LP Model Applications in Open Pit Mining

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Outline

- Introduction
- Linear Programming (LP)
- Applications in Mining Industry
- Example Problem1: Optimal Route Selection in Open Pit Mining (Truck Dispatching Problem)
- Example Problem2: Ultimate Pit Limit by Positive Moving Cone
- Example Problme3: Cutoff Grade Optimization Problem

Introduction

- **Mathematical Programming:** A mathematical programming model is a decision model for planning decisions that optimize an objective function and satisfy limitations imposed by mathematical constraints .
- **Optimization:** Finding a solution which results in either the minimum cost or maximum performance under the given set of conditions .

Linear Programming (LP)

- In mathematical notation, a linear programming problem is expressed as follows:

Max (Min) **$Z = C X$** **(objective function)**

Subject to :

$A X \le\ge b$ **(set of constraints)**

$X \ge 0$ **(non-negativity)**

Where
C: row vector of objective function coefficients,
X: column vector of decisions variables,
A: matrix of constraints coefficients,
b: column vector of right hand side values, representing available resources

Linear Programming (LP) Continued

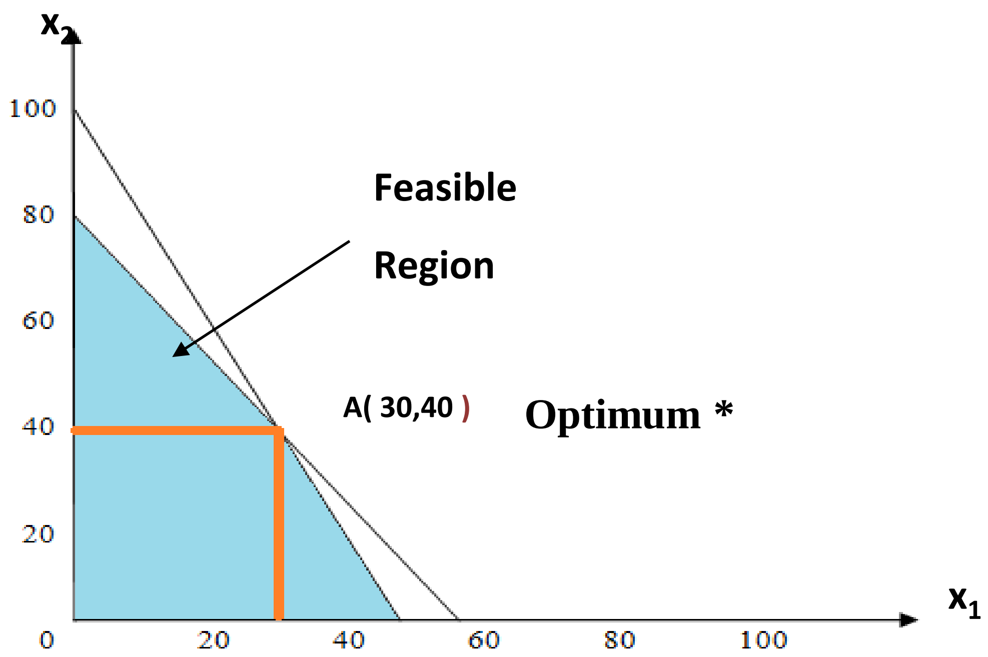
Max z= $7x_1 + 5x_2$

$4x_1 + 3x_2 \leq 240$

$2x_1 + x_2 \leq 100$

$x_1, x_2 \geq 0$

Linear Programming (LP) Example –Graphical Solution

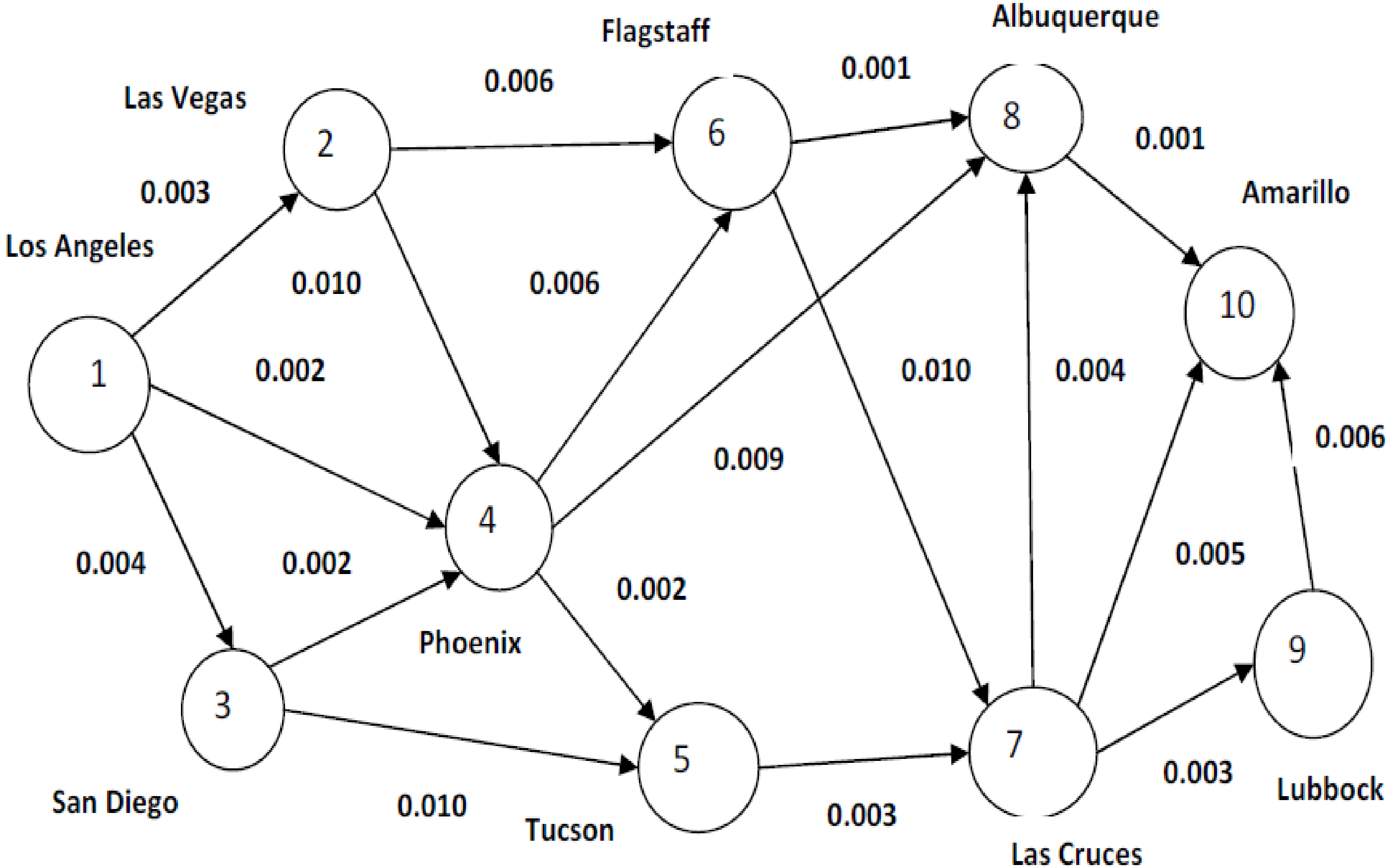


Applications in Mining Industry

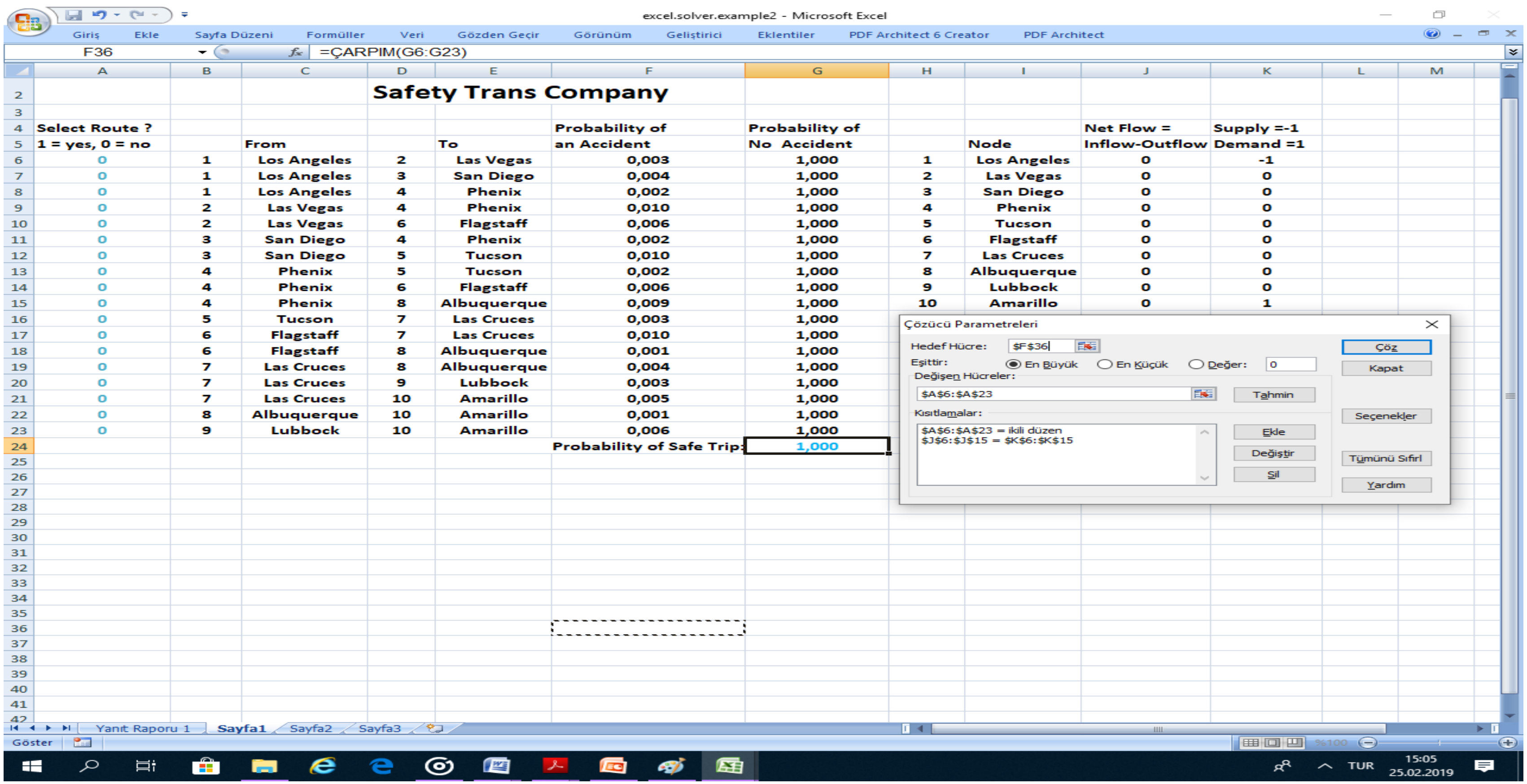
- Determining Ultimate Pit limits,
- Finding the optimal sequence of blocks extraction,
- Finding the optimal blending pattern,
- Cutoff grade optimization,
- Finding the optimal places for blasting,
- Truck Dispatching in open pit
- Equipment Replacements
- and many more..

Network Diagram for Safety Trans Company (STC)

–



Spreadsheet and Excel Solver Model for STC



Safety Trans Company

Select Route ? 1 = yes, 0 = no	From		To		Probability of an Accident	Probability of No Accident	Node		Net Flow = Inflow-Outflow	Supply =-1 Demand =1
0	1	Los Angeles	2	Las Vegas	0,003	1,000	1	Los Angeles	0	-1
0	1	Los Angeles	3	San Diego	0,004	1,000	2	Las Vegas	0	0
0	1	Los Angeles	4	Phenix	0,002	1,000	3	San Diego	0	0
0	2	Las Vegas	4	Phenix	0,010	1,000	4	Phenix	0	0
0	2	Las Vegas	6	Flagstaff	0,006	1,000	5	Tucson	0	0
0	3	San Diego	4	Phenix	0,002	1,000	6	Flagstaff	0	0
0	3	San Diego	5	Tucson	0,010	1,000	7	Las Cruces	0	0
0	4	Phenix	5	Tucson	0,002	1,000	8	Albuquerque	0	0
0	4	Phenix	6	Flagstaff	0,006	1,000	9	Lubbock	0	0
0	4	Phenix	8	Albuquerque	0,009	1,000	10	Amarillo	0	1
0	5	Tucson	7	Las Cruces	0,003	1,000				
0	6	Flagstaff	7	Las Cruces	0,010	1,000				
0	6	Flagstaff	8	Albuquerque	0,001	1,000				
0	7	Las Cruces	8	Albuquerque	0,004	1,000				
0	7	Las Cruces	9	Lubbock	0,003	1,000				
0	7	Las Cruces	10	Amarillo	0,005	1,000				
0	8	Albuquerque	10	Amarillo	0,001	1,000				
0	9	Lubbock	10	Amarillo	0,006	1,000				
					Probability of Safe Trip:	1,000				

Spreadsheet and Optimal Solver Output for STC

excel.solver.example2 - Microsoft Excel

G24 =ÇARPIM(G6:G23)

Safety Trans Company

Select Route ? 1 = yes, 0 = no		From		To	Probability of an Accident	Probability of No Accident		Node	Net Flow = Inflow-Outflow	Supply =-1 Demand =1
0	1	Los Angeles	2	Las Vegas	0,003	1,000	1	Los Angeles	-1	-1
0	1	Los Angeles	3	San Diego	0,004	1,000	2	Las Vegas	0	0
1	1	Los Angeles	4	Phenix	0,002	0,998	3	San Diego	0	0
0	2	Las Vegas	4	Phenix	0,010	1,000	4	Phenix	0	0
0	2	Las Vegas	6	Flagstaff	0,006	1,000	5	Tucson	0	0
0	3	San Diego	4	Phenix	0,002	1,000	6	Flagstaff	0	0
0	3	San Diego	5	Tucson	0,010	1,000	7	Las Cruces	0	0
0	4	Phenix	5	Tucson	0,002	1,000	8	Albuquerque	0	0
1	4	Phenix	6	Flagstaff	0,006	0,994	9	Lubbock	0	0
0	4	Phenix	8	Albuquerque	0,009	1,000	10	Amarillo	1	1
0	5	Tucson	7	Las Cruces	0,003	1,000				
0	6	Flagstaff	7	Las Cruces	0,010	1,000				
1	6	Flagstaff	8	Albuquerque	0,001	0,999				
0	7	Las Cruces	8	Albuquerque	0,004	1,000				
0	7	Las Cruces	9	Lubbock	0,003	1,000				
0	7	Las Cruces	10	Amarillo	0,005	1,000				
1	8	Albuquerque	10	Amarillo	0,001	0,999				
0	9	Lubbock	10	Amarillo	0,006	1,000				
					Probability of Safe Trip:	0,990				

Yanıt Raporu 1 | Yanıt Raporu 2 | Sayfa1 | Sayfa2 | Sayfa3

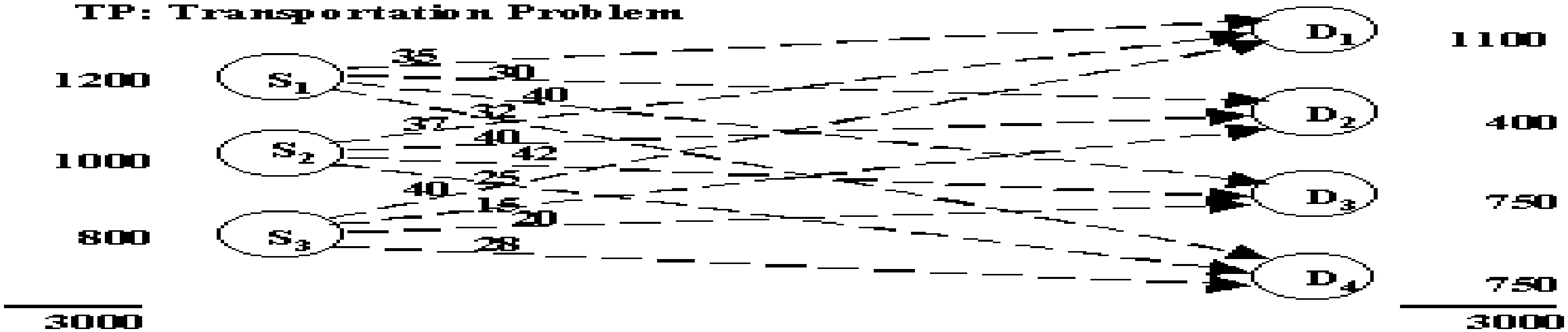
Network Models

Network Models

Presentation of a Network Problem by:

- A set of nodes
- A set of arcs
- A cost function for each arc

TP: Transportation Problem



LP Formulation:

Min $35X_{11} + 30X_{12} + 40X_{13} + 32X_{14} + 37X_{21} + 40X_{22} + 42X_{23} + 25X_{24} + 40X_{31} + 15X_{32} + 20X_{33} + 28X_{34}$

subject to

$$X_{11} + X_{12} + X_{13} + X_{14} \le 1200$$
$$X_{21} + X_{22} + X_{23} + X_{24} \le 1000$$
$$X_{31} + X_{32} + X_{33} + X_{34} \le 800$$
$$X_{11} + X_{21} + X_{31} \ge 1100$$
$$X_{12} + X_{22} + X_{32} \ge 400$$
$$X_{13} + X_{23} + X_{33} \ge 750$$
$$X_{14} + X_{24} + X_{34} \ge 750$$
$$X_{ij} \ge 0$$

LINDO Model for Transportation Problem

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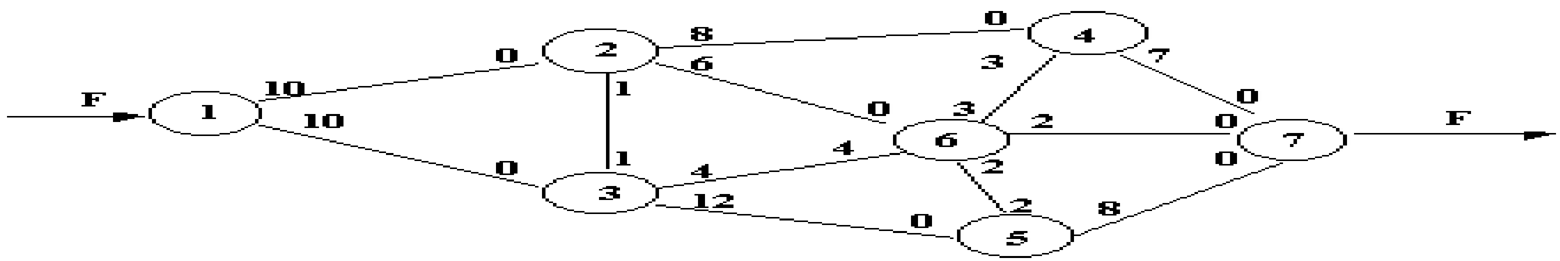
```
!
!   LP Model for Transportation Problem
!
! X<ij> =  Denote the flow along arc (i to j) by Xij.
!
Min   35 X11 + 30 X12 + 40 X13 + 32 X14 + 37 X21  + 40 X22 + 42 X23 + 25 X24
    + 40 X31 + 15 X32 + 20 X33 + 28 X34

SUBJECT TO
!
!   Demand constraints:
C1)   X11 + X12 + X13 + X14        <=    1200
C2)   X21 + X22 + X23 + X24        <=    1000
C3)   X31 + X32 + X33 + X34        <=    800
C4)   X11 + X21 + X31              >=    1100
C5)   X12 + X22 + X32              >=    400
C6)   X13 + X23 + X33              >=    750
C7)   X14 + X24 + X34              >=    750
C8)   X11 >= 0
C9)   X12 >= 0
C10)  X13 >= 0
C11)  X14 >= 0
C12)  X21 >= 0
C13)  X22 >= 0
C14)  X23 >= 0
C15)  X24 >= 0
C16)  X31 >= 0
C17)  X32 >= 0
C18)  X33 >= 0
C19)  X34 >= 0
C20)  X41 >= 0
C21)  X42 >= 0
C22)  X43 >= 0
C23)  X44 >= 0

END
!
```

LINDO Output for Transportation Problem

```
LINDO - [Reports Window]
File   Edit   Solve   Reports   Window   Help

LP OPTIMUM FOUND AT STEP        6

       OBJECTIVE FUNCTION VALUE

       1)        84000.00

 VARIABLE        VALUE          REDUCED COST
      X11       850.000000          0.000000
      X12       350.000000          0.000000
      X13         0.000000          5.000000
      X14         0.000000          9.000000
      X21       250.000000          0.000000
      X22         0.000000          8.000000
      X23         0.000000          5.000000
      X24       750.000000          0.000000
      X31         0.000000         20.000000
      X32        50.000000          0.000000
      X33       750.000000          0.000000
      X34         0.000000         20.000000
      X41         0.000000          0.000000
      X42         0.000000          0.000000
      X43         0.000000          0.000000
      X44         0.000000          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
      C1)         0.000000          2.000000
      C2)         0.000000          0.000000
      C3)         0.000000         17.000000
      C4)         0.000000        -37.000000
      C5)         0.000000        -32.000000
      C6)         0.000000        -37.000000
      C7)         0.000000        -25.000000
      C8)       850.000000          0.000000
      C9)       350.000000          0.000000
     C10)         0.000000          0.000000
     C11)         0.000000          0.000000
     C12)       250.000000          0.000000
     C13)         0.000000          0.000000
     C14)         0.000000          0.000000
     C15)       750.000000          0.000000
     C16)         0.000000          0.000000
     C17)        50.000000          0.000000
     C18)       750.000000          0.000000
     C19)         0.000000          0.000000
     C20)         0.000000          0.000000
     C21)         0.000000          0.000000
     C22)         0.000000          0.000000
     C23)         0.000000          0.000000

NO. ITERATIONS=        6
```

Network Models – Max Flow Problems

Max Flow Problem



max F

subject to:

Origin: $X_{12} + X_{13} - F = 0$

Intermediate Nodes:

$X_{12} + X_{32} - X_{23} - X_{26} - X_{24} = 0$

$X_{13} + X_{23} + X_{63} - X_{32} - X_{36} - X_{35} = 0$

$X_{24} + X_{64} - X_{47} - X_{46} = 0$

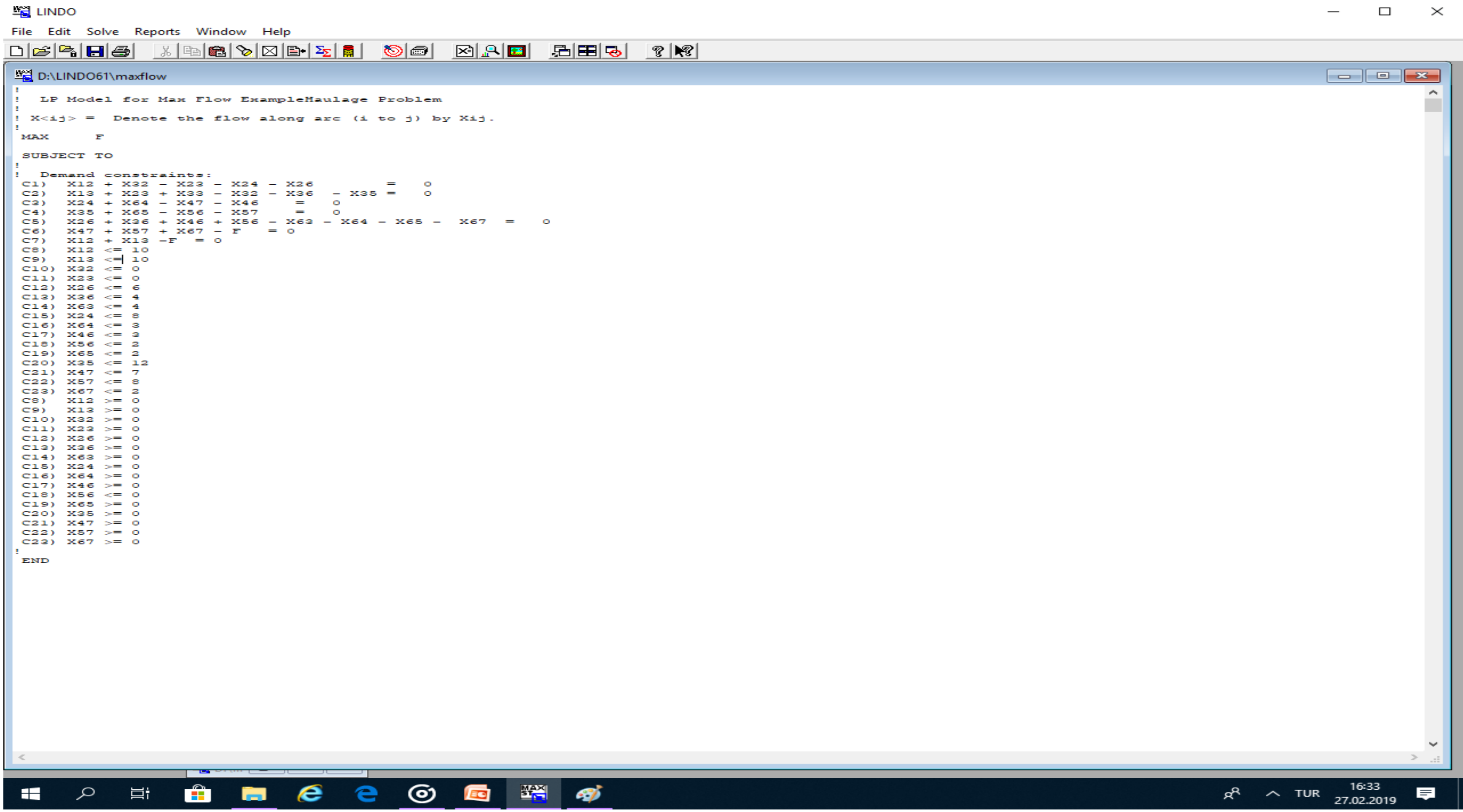
$X_{35} + X_{65} - X_{56} - X_{57} = 0$

$X_{26} + X_{46} + X_{36} + X_{56} - X_{65} - X_{63} - X_{64} - X_{67} = 0$

Destination: $X_{47} + X_{67} + X_{57} - F = 0$

$X_{12} \leq 10$	$X_{64} \leq 3$
$X_{13} \leq 10$	$X_{46} \leq 3$
$X_{23} \leq 1$	$X_{35} \leq 12$
$X_{32} \leq 1$	$X_{65} \leq 2$
$X_{26} \leq 6$	$X_{56} \leq 2$
$X_{36} \leq 4$	$X_{57} \leq 8$
$X_{63} \leq 4$	$X_{47} \leq 7$
$X_{24} \leq 8$	$X_{67} \leq 2$
$X_{ij} \geq 0$	

LINDO Model for Max Flow Problem

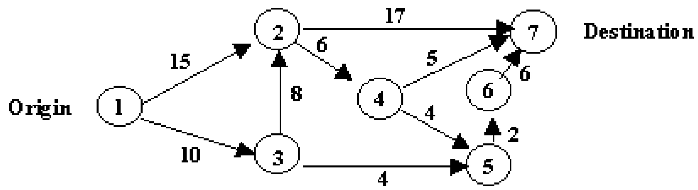


LINDO Output for Max Flow Problem

```
LP OPTIMUM FOUND AT STEP      10

       OBJECTIVE FUNCTION VALUE

       1)        17.00000

 VARIABLE          VALUE            REDUCED COST
        F        17.000000            0.000000
      X12        10.000000            0.000000
      X32         0.000000            0.000000
      X23         0.000000            0.000000
      X24         7.000000            0.000000
      X26         3.000000            0.000000
      X13         7.000000            0.000000
      X33         1.000000            0.000000
      X36         0.000000            0.000000
      X35         8.000000            0.000000
      X64         0.000000            0.000000
      X47         7.000000            0.000000
      X46         0.000000            0.000000
      X65         0.000000            0.000000
      X56         0.000000            0.000000
      X57         8.000000            0.000000
      X63         1.000000            0.000000
      X67         2.000000            0.000000
```

Network Models – Shortest Path Problems

Shortest Path Problem



$$\min \sum_{\text{all arcs}} C_{ij}X_{ij}$$

$\sum_{\text{arcs out}} X_{ij} - \sum_{\text{arcs in}} X_{ij} = 1$ Origin Node i

$\sum_{\text{arcs out}} X_{ij} - \sum_{\text{arcs in}} X_{ij} = 0$ Intermediate Nodes

$\sum_{\text{arcs in}} X_{ij} - \sum_{\text{arcs out}} X_{ij} = 1$ Destination Node

$X_{ij} \geq 0$

For unacceptable routes add new constraint $X_{ij} = 0$

Min $15X_{12} + 10X_{13} + 8X_{32} + 6X_{24} + 17X_{27} + 4X_{35} + 5X_{47} + 4X_{45} + 2X_{56} + 6X_{67}$

subject to

$X_{12} + X_{13} = 1$

$X_{12} + X_{32} - X_{24} - X_{27} = 0$

$X_{13} - X_{32} - X_{35} = 0$

$X_{24} - X_{47} - X_{45} = 0$

$X_{35} + X_{45} - X_{56} = 0$

$X_{56} - X_{67} = 0$

$X_{27} + X_{47} + X_{67} = 1$

LINDO Model for Shortest Path Problem

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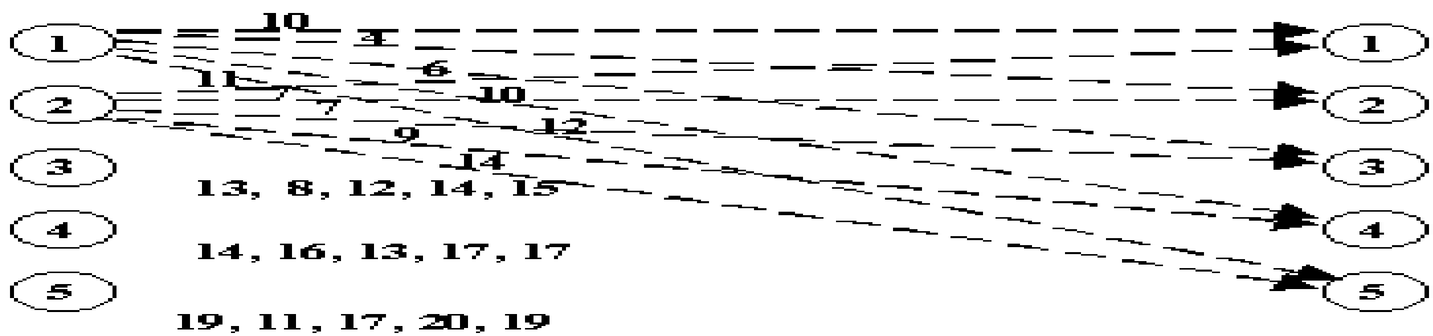
```
!
!   LP Model for Shortest Path Example Problem
!
! X<ij> =  Denote the cost along arc (i to j) by Xij.
!
Min  15 X12 + 10 X13 + 8 X32 + 6 X24 + 17 X27 + 4 X35 + 5 X47 + 4 X45 + 2 X56 + 6 X67

SUBJECT TO
!
!   Demand constraints:
C1)   X12 + X13              =    1
C2)   X12 + X32 - X24   - X27 =    0
C3)   X13 - X32 - X35     =    0
C4)   X24 - X47 - X45     =    0
C5)   X35 + X45 -  X56    =    0
C6)   X56 -  X67    = 0
C7)   X27 + X47 + X67   = 1
C8)   X12  >=   0
C9)   X13  >=   0
C10)  X32 >= 0
C15)  X24 >= 0
C15)  X27 >= 0
C17)  X45 >= 0
C18)  X56 >= 0
C19)  X65 >= 0
C20)  X35 >= 0
C21)  X47 >= 0
C23)  X67 >= 0
!
END
```

LINDO Output for Shortest Path Problem

```
LP OPTIMUM FOUND AT STEP        5

        OBJECTIVE FUNCTION VALUE

        1)       22.00000

 VARIABLE         VALUE          REDUCED COST
      X12        0.000000          0.000000
      X13        1.000000          0.000000
      X32        0.000000          3.000000
      X24        0.000000          0.000000
      X27        0.000000         10.000000
      X35        1.000000          0.000000
      X47        0.000000          4.000000
      X45        0.000000         11.000000
      X56        1.000000          0.000000
      X67        1.000000          0.000000
      X65        0.000000          0.000000
```

Network Models – Assignment Problems

Assignment Problem



LP Formulation:

Min $10X_{11} + 4X_{12} + 6X_{13} + 10X_{14} + 12X_{15} + 11X_{21} + 7X_{22} + 7X_{23} + 9X_{24} + 14X_{25}$
$+ 13X_{31} + 8X_{32} + 12X_{33} + 14X_{34} + 15X_{35} + 14X_{41} + 16X_{42} + 13X_{43} + 17X_{44} +$
$17X_{45} + 19X_{51} + 11X_{52} + 17X_{53} + 20X_{54} + 19X_{55}$

subject to

$$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} = 1$$
$$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} = 1$$
$$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} = 1$$
$$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} = 1$$
$$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} = 1$$
$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} = 1$$
$$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} = 1$$
$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} = 1$$
$$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} = 1$$
$$X_{15} + X_{25} + X_{35} + X_{45} + X_{45} = 1$$
$$X_{ij} \geq 0$$

LINDO Model for Assignment Problem

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File Edit Solve Reports Window Help

```
!
!  LP Model for Assignment Problem
!
! X<ij> =  Denote the flow along arc (i to j) by Xij.
!
Min      10 X11 + 4 X12 + 6 X13 + 10 X14 + 12 X15  + 11 X21 + 7 X22 + 7 X23 + 9 X24 + 14 X25 +
     13 X31 + 8 X32 + 12 X33 + 14 X34 + 15 X35 + 14 X41 + 16 X42 + 13 X43 + 17 X44 + 17 X45 +
     19 X51 + 11 X52 + 17 X53 + 20 X54 + 19 X55

SUBJECT TO
!
!
C1)  X11 + X12 + X13 + X14 + X15     =   1
C2)  X21 + X22 + X23 + X24 + X25     =   1
C3)  X31 + X32 + X33 + X34 + X35     =   1
C2)  X41 + X42 + X43 + X44 + X45     =   1
C3)  X51 + X52 + X53 + X54 + X55     =   1
C4)  X11 + X21 + X31 + X41 + X51     =   1
C5)  X12 + X22 + X32 + X42 + X52     =   1
C6)  X13 + X23 + X33 + X43 + X53     =   1
C7)  X14 + X24 + X34 + X44 + X54     =   1
C8)  X15 + X25 + X35 + X45 + X55     =   1
C8)  X11 >= 0
C9)  X12 >= 0
C10) X13 >= 0
C11) X14 >= 0
C11) X15 >= 0
C12) X21 >= 0
C13) X22 >= 0
C14) X23 >= 0
C15) X24 >= 0
C11) X25 >= 0
C11) X31 >= 0
C17) X32 >= 0
C18) X33 >= 0
C19) X34 >= 0
C11) X35 >= 0
C20) X41 >= 0
C21) X42 >= 0
C22) X43 >= 0
C23) X44 >= 0
C11) X45 >= 0
C20) X51 >= 0
C21) X52 >= 0
C22) X53 >= 0
C23) X54 >= 0
C24) X55 >= 0

 END

INT X11
INT X12
INT X13
INT X14
INT X15

INT X21
INT X22
INT X23
INT X24
INT X25

INT X31
INT X32
INT X33
INT X34
INT X35

INT X41
INT X42
INT X43
INT X44
INT X45
```

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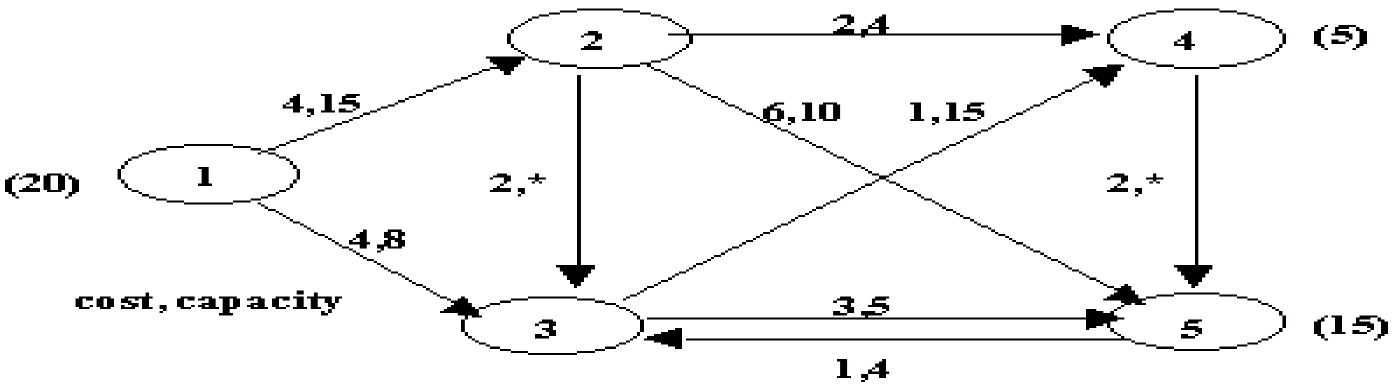
LINDO Output for Assignment Problem

Reports Window

```
LP OPTIMUM FOUND AT STEP      14
OBJECTIVE VALUE =    55.0000000


NEW INTEGER SOLUTION OF    55.0000000      AT BRANCH       0 PIVOT       14
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)       55.00000

 VARIABLE         VALUE          REDUCED COST
      X11         0.000000          10.000000
      X12         0.000000           4.000000
      X13         1.000000           6.000000
      X14         0.000000          10.000000
      X15         0.000000          12.000000
      X21         0.000000          11.000000
      X22         0.000000           7.000000
      X23         0.000000           7.000000
      X24         1.000000           9.000000
      X25         0.000000          14.000000
      X31         0.000000          13.000000
      X32         0.000000           8.000000
      X33         0.000000          12.000000
      X34         0.000000          14.000000
      X35         1.000000          15.000000
      X41         1.000000          14.000000
      X42         0.000000          16.000000
      X43         0.000000          13.000000
      X44         0.000000          17.000000
      X45         0.000000          17.000000
      X51         0.000000          19.000000
      X52         1.000000          11.000000
      X53         0.000000          17.000000
      X54         0.000000          20.000000
      X55         0.000000          19.000000
```

Network Models – Min Cost Problems

Min Cost Flow



min $4X_{12} + 4X_{13} + 2X_{23} + 2X_{24} + 6X_{25} + 1X_{34} + 3X_{35} + 2X_{45} + X_{53}$

subject to:

$X_{12} + X_{13} \leq 20$	Source node
$X_{12} - X_{24} - X_{25} - X_{23} = 0$	Tranship node
$X_{13} + X_{23} + X_{53} - X_{34} - X_{35} = 0$	
$X_{24} + X_{34} - X_{45} = 5$	Sink node
$X_{35} + X_{25} + X_{45} - X_{53} = 15$	Sink node

$X_{12} \leq 15$	$X_{34} \leq 15$
$X_{13} \leq 8$	$X_{25} \leq 10$
$X_{35} \leq 5$	$X_{53} \leq 4$
$X_{24} \leq 4$	$X_{25} \leq 10$

$X_{ij} \geq 0$

LINDO Model for Min Cost Problem

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```
!
!   LP Model for Min Cost Problem
!
! X<ij> =  Denote the flow along arc (i to j) by Xij.
!
 Min  4 X12 + 4 X13 + 2 X23 + 2 X24 + 6 X25  + 1 X34 + 3 X35 + 2 X45 + 1 X53

 SUBJECT TO
!
!   Demand constraints:
 C1)   X12 + X13        <=   20
 C2)   X12 - X23 - X24  - X25 =    0
 C5)   X13 + X23 + X53 - X34 -  X35  =    0
 C6)   X24 + X34 - X45    = 5
 C7)   X25 + X35 + X45 - X53   = 15
 C8)   X12 <= 15
 C9)   X13 <= 8
 C10)  X35 <= 5
 C11)  X24 <= 4
 C12)  X34 <= 15
 C13)  X52 <= 10
 C14)  X53 <= 4
 C15)  X25 <= 10
 C16)  X12 >= 0
 C17)  X13 >= 0
 C18)  X35 >= 0
 C19)  X24 >= 0
 C20)  X34 >= 0
 C22)  X53 >= 0
 C23)  X25 >= 0
 C24)  X23 >= 0
 C25)  X45 >= 0
 END
!
```

LINDO Output for min Cost Problem

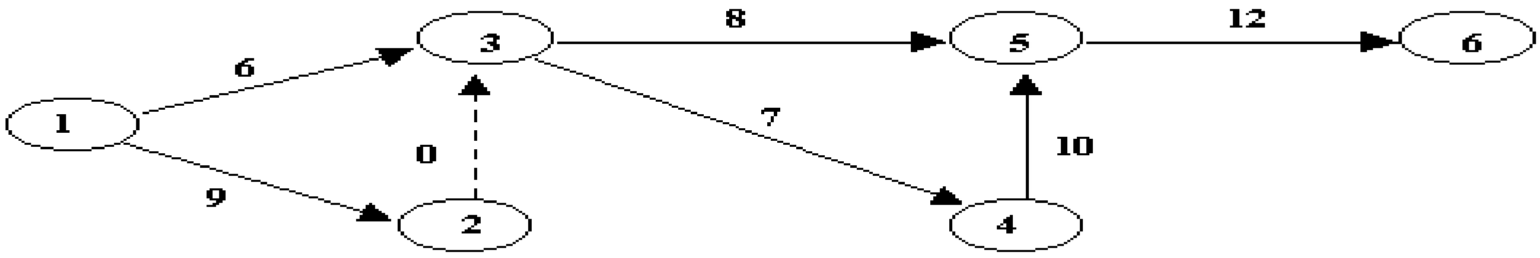
LINDO - [Reports Window]

File Edit Solve Reports Window Help

```
LP OPTIMUM FOUND AT STEP        7

       OBJECTIVE FUNCTION VALUE

       1)        150.0000

  VARIABLE        VALUE          REDUCED COST
       X12        12.000000          0.000000
       X13         8.000000          0.000000
       X23         8.000000          0.000000
       X24         4.000000          0.000000
       X25         0.000000          1.000000
       X34        15.000000          0.000000
       X35         1.000000          0.000000
       X45        14.000000          0.000000
       X53         0.000000          4.000000
       X52         0.000000          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
       C1)         0.000000          0.000000
       C2)         0.000000         -4.000000
       C5)         0.000000         -6.000000
       C6)         0.000000         -7.000000
       C7)         0.000000         -9.000000
       C8)         3.000000          0.000000
       C9)         0.000000          2.000000
      C10)         4.000000          0.000000
      C11)         0.000000          1.000000
      C12)         0.000000          0.000000
      C13)        10.000000          0.000000
      C14)         4.000000          0.000000
      C15)        10.000000          0.000000
      C16)        12.000000          0.000000
      C17)         8.000000          0.000000
      C18)         1.000000          0.000000
      C19)         4.000000          0.000000
      C20)        15.000000          0.000000
      C22)         0.000000          0.000000
      C23)         0.000000          0.000000
      C24)         8.000000          0.000000
      C25)        14.000000          0.000000

NO. ITERATIONS=        7
```

Network Models – Critical Path Problems

Critical Path Method



max		$9X_{12} + 6X_{13} + 8X_{35} + 7X_{34} + 10X_{45} + 12X_{56}$
subject to:		
starting node	$X_{12} + X_{13} = 1$	
intermediate nodes		$X_{12} - X_{23} = 0$ $X_{13} + X_{23} - X_{34} - X_{35} = 0$ $X_{34} - X_{45} = 0$ $X_{35} + X_{45} - X_{56} = 0$
finish node		$x_{56} = 1$
		$x_{ij} \geq 0$

LINDO Model for Critical Path Problem

LINDO - [D:\LINDO61\critical_path1.txt]

File Edit Solve Reports Window Help

```
 LP Model for Critical Path Problem

X<ij> =  Denote the flow along arc (i to j) by Xij.

MAX   9 X12 + 6 X13 + 8 X35 + 7 X34 + 10 X45  + 12 X56

SUBJECT TO


C1)  X12 + X13                   =   1
C2)  X12 - X23                   =   0
C3)  X13 + X23 - X34 - X35       =   0
C4)  X34 - X45                   =   0
C5)  X35 + X45  - X56            =   0
C6)  X56                         =   1
C7)  X12 >= 0
C8)  X35 >= 0
C9)  X34 >= 0
C10) X45 >= 0
C11) X56 >= 0

END
```

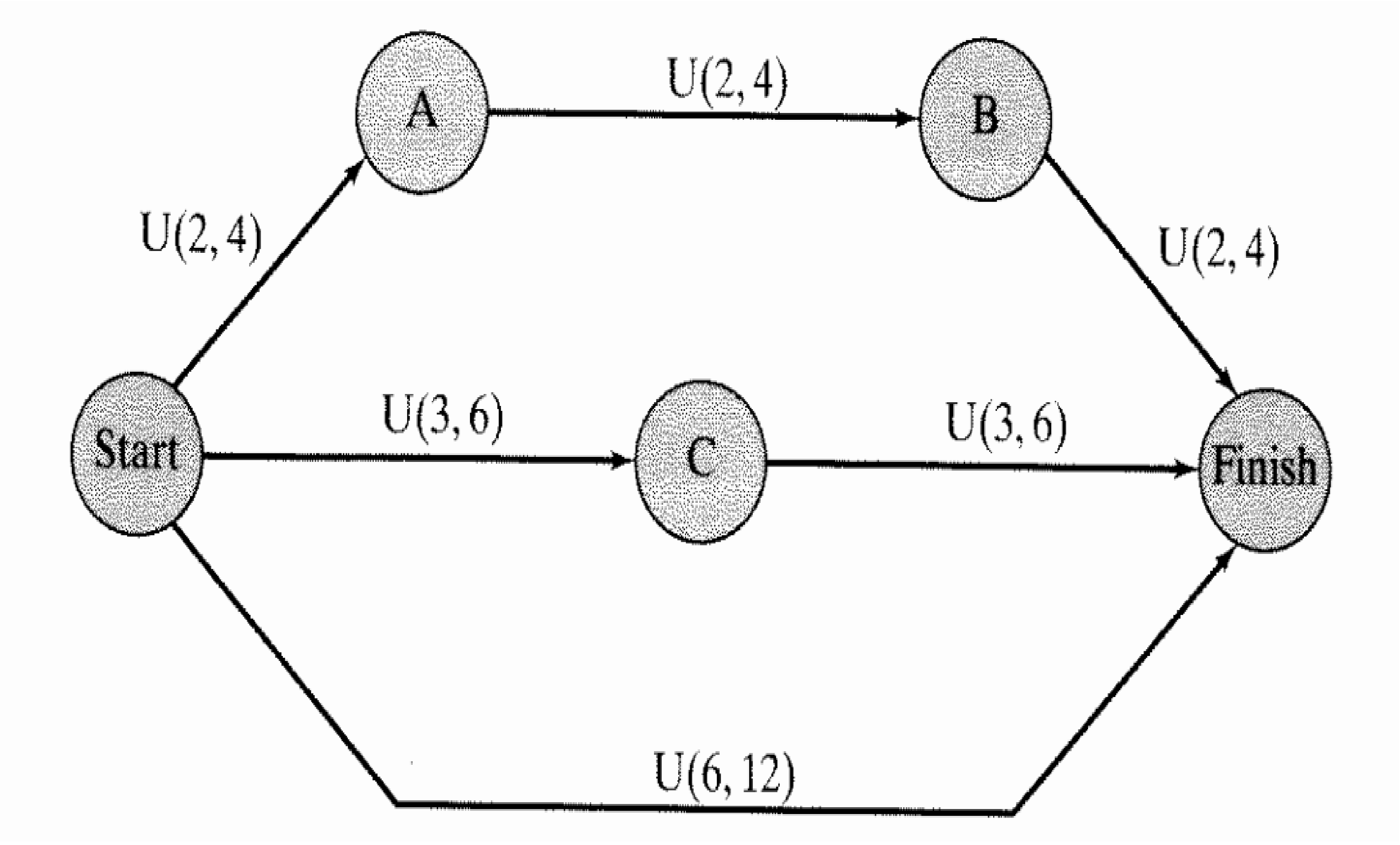
LINDO Output for Critical Path Problem

LINDO - [Reports Window]

File Edit Solve Reports Window Help

```
LP OPTIMUM FOUND AT STEP        2

       OBJECTIVE FUNCTION VALUE

       1)       38.00000

 VARIABLE        VALUE          REDUCED COST
      X12         1.000000          0.000000
      X13         0.000000          3.000000
      X35         0.000000          9.000000
      X34         1.000000          0.000000
      X45         1.000000          0.000000
      X56         1.000000          0.000000
      X23         1.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
      C1)         0.000000          9.000000
      C2)         0.000000          0.000000
      C3)         0.000000          0.000000
      C4)         0.000000          7.000000
      C5)         0.000000         17.000000
      C6)         0.000000         29.000000
      C7)         1.000000          0.000000
      C8)         0.000000          0.000000
      C9)         1.000000          0.000000
```

Activity Network for Example Problem.



Project Simulation Example (An Activity Network)

Project Simulation Example (An Activity Network)

Click the button to recalculate the spreadsheet and generate a new trial.

Generate New Trial

Paths	Activities	Lower Limit	Upper Limit
1	1	2	4
	2	2	4
	3	2	4
2	1	3	6
	2	3	6
3	3	6	12

You may change the Example by changing any of the values in green.

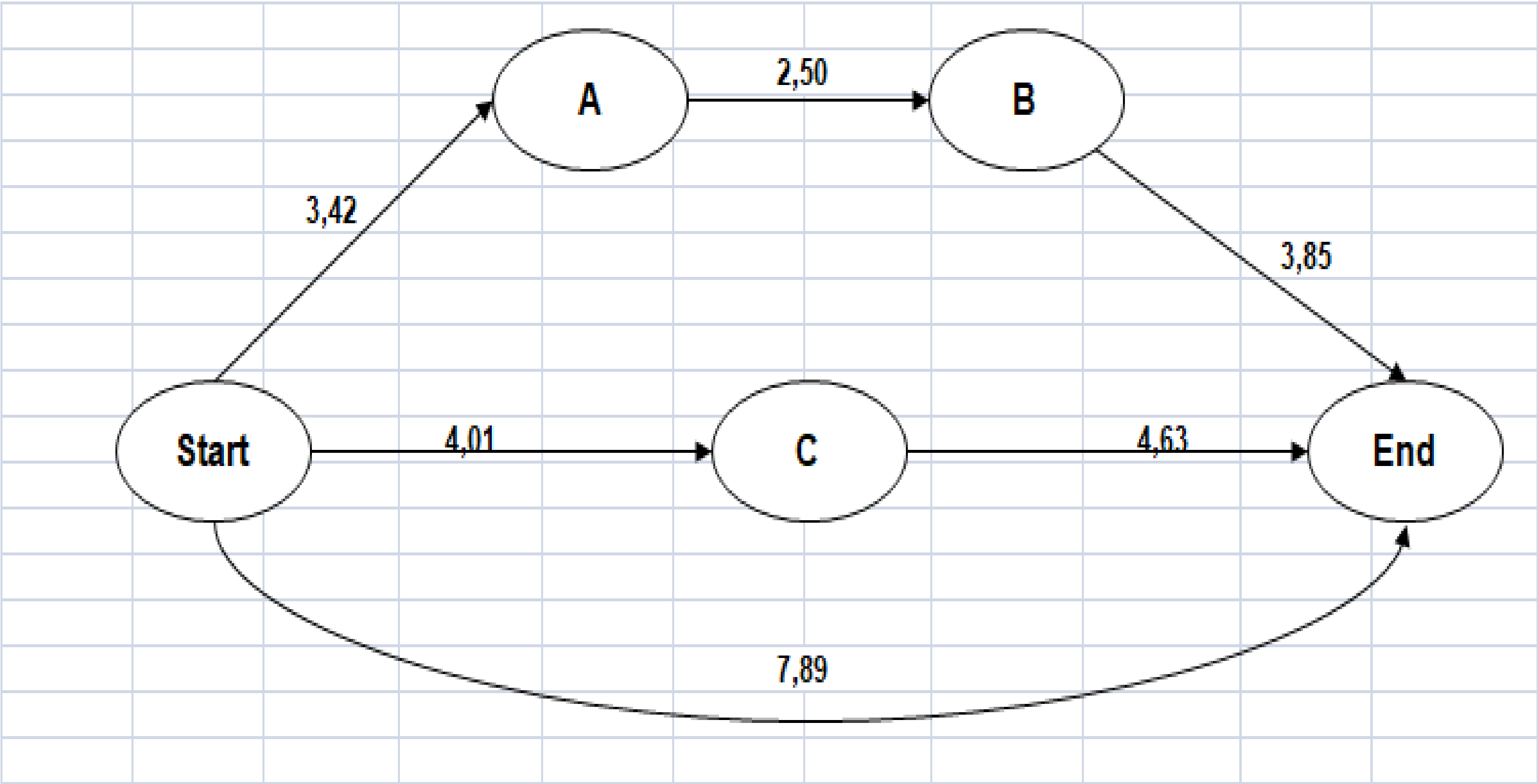
Seed for Random Numbers 12345 — Reset Seed & Run

Number of Customers = 100

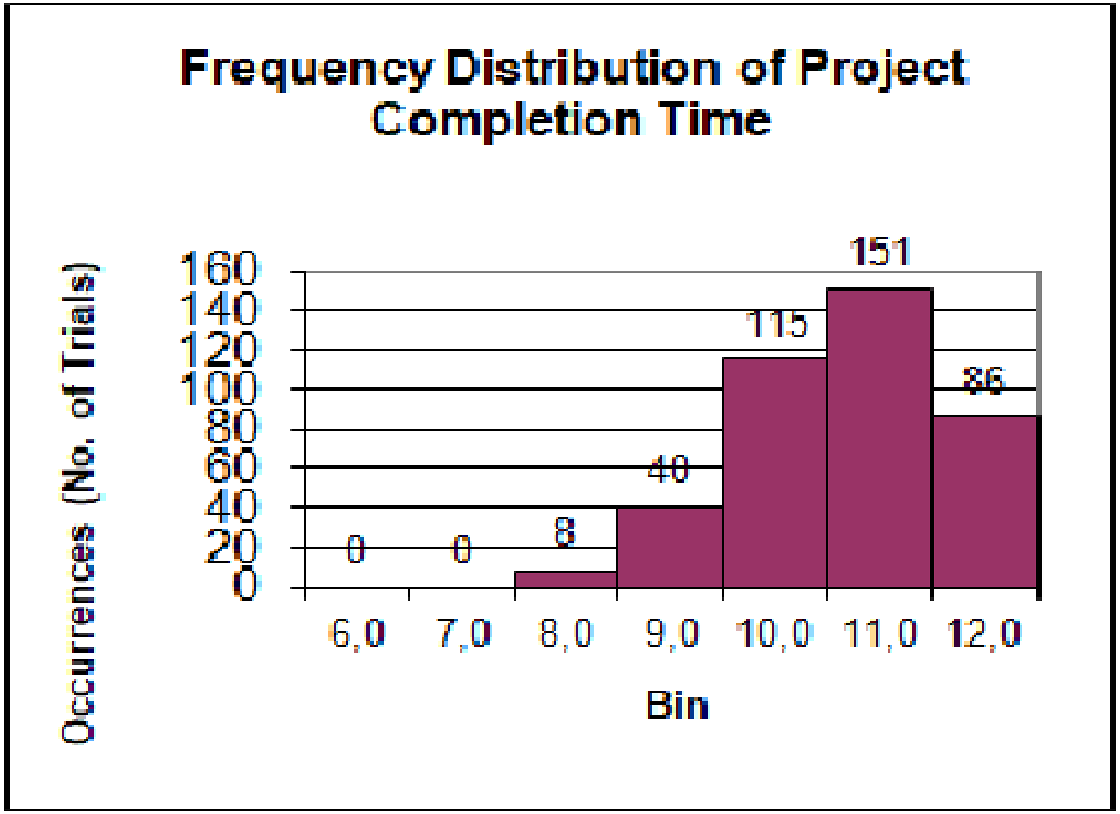
Simulation Table

Path 1			Path 2		Path 3	Completion Times				Critical Path
Activity 1	Activity 2	Activity 3	Activity 1	Activity 2	Activity 1	Path 1	Path 2	Path 3	Project	
3,42	2,50	3,85	4,01	4,63	7,89	9,77	8,64	7,89	9,77	1

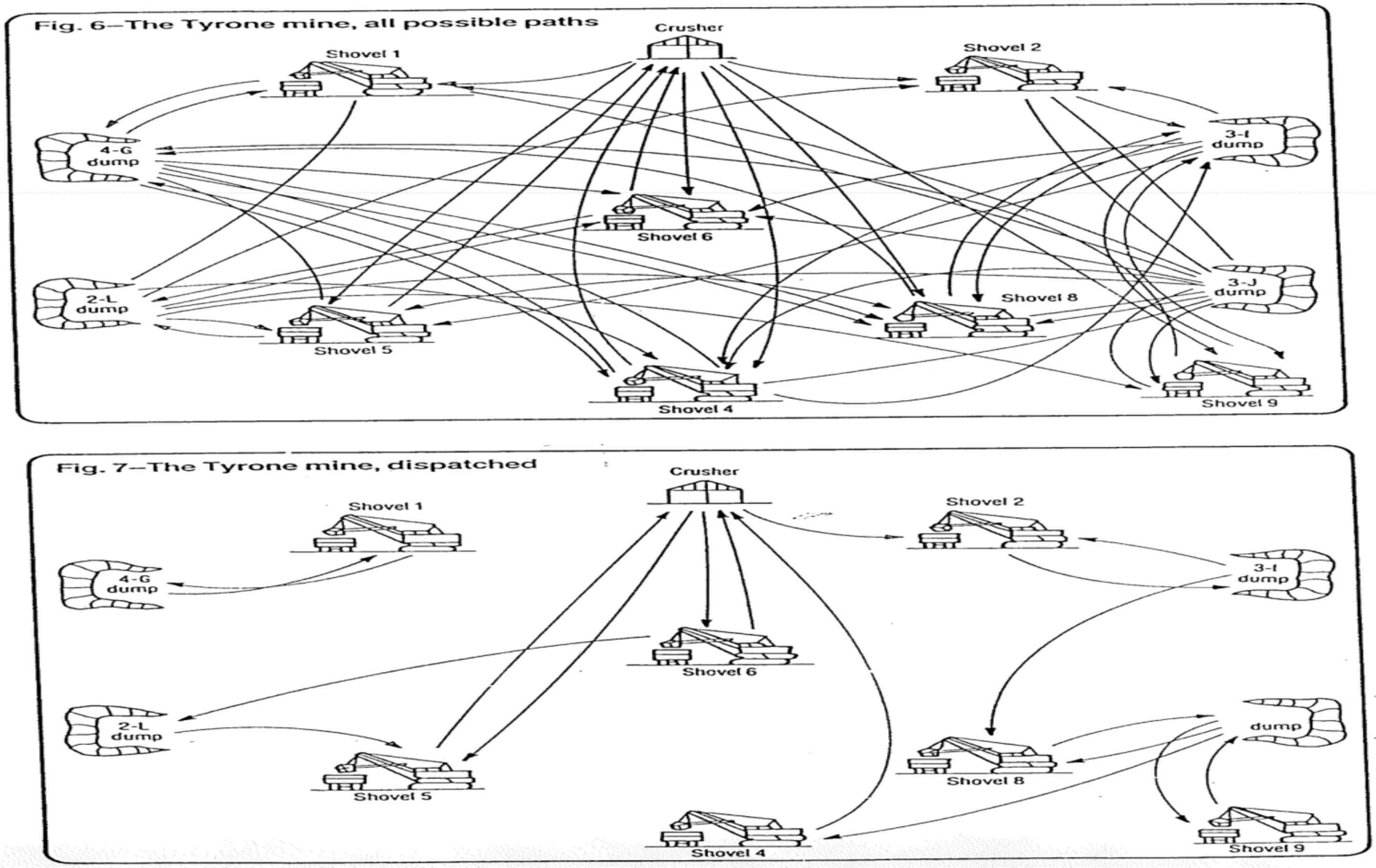
One Simulation Run for Example Problem



Frequency Distribution for Project Completion Time



Example Problem 1: Tyrone Mine Paper



Open Pit Truck/Shovel Haulage System Optimization (Optimal Route Selection, Four-node example, all possible paths

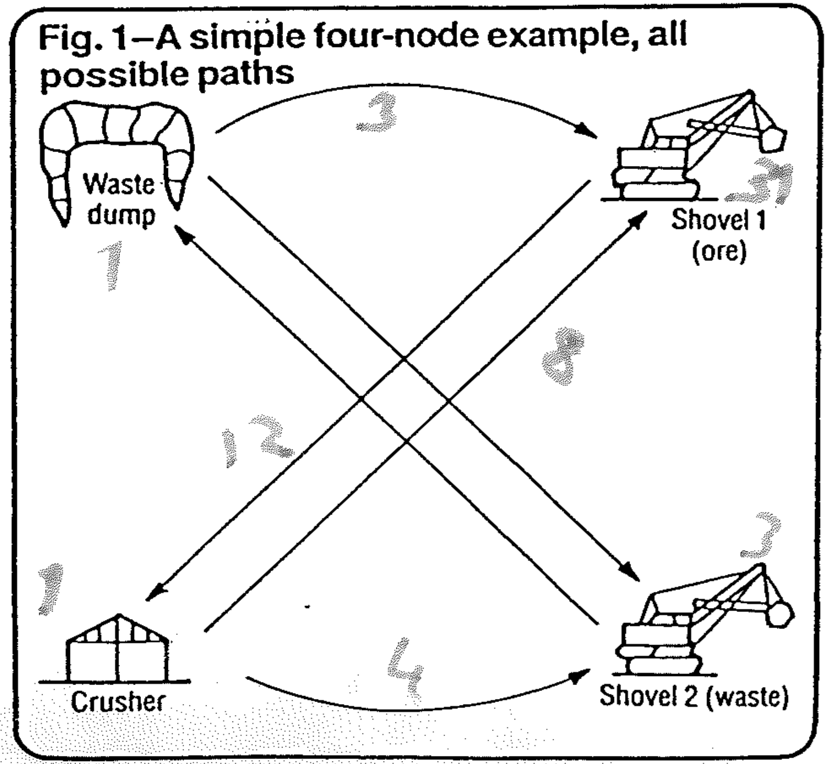
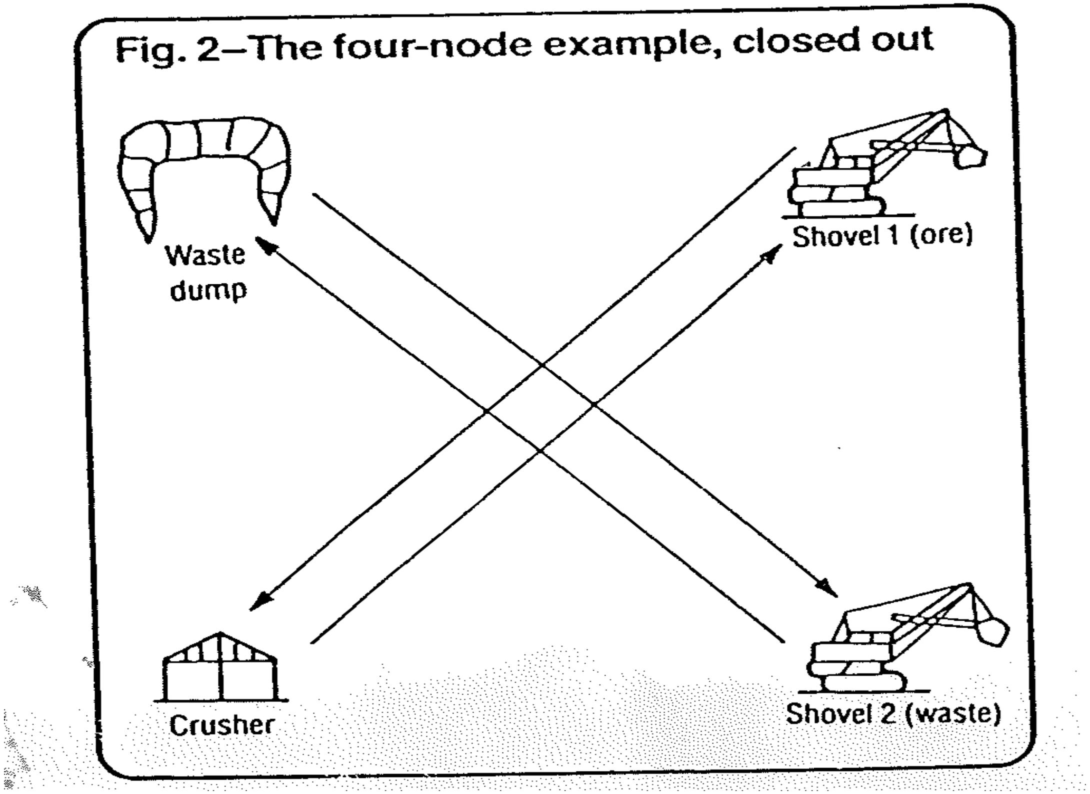


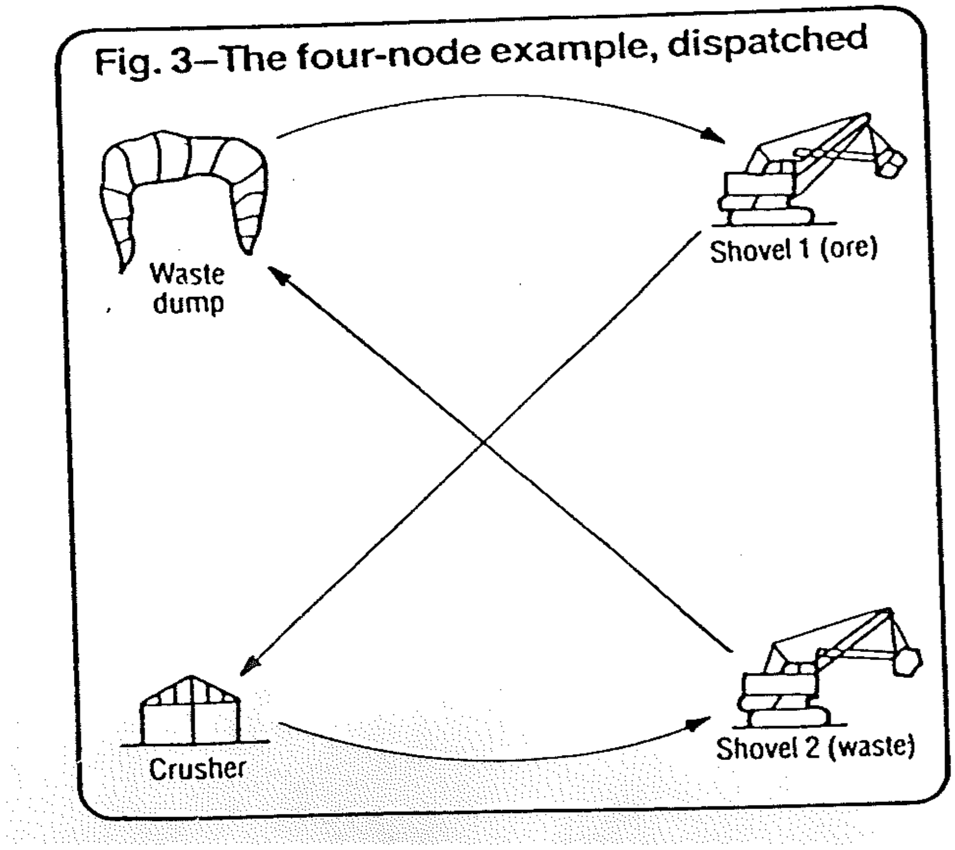
Table 1 – Four-node pit travel and transaction times

Shovel 1 to crusher	12 min
Crusher to shovel 1	8 min
Shovel 2 to dump	12 min
Waste dump to shovel	8 min
Crusher to shovel 2	4 min
Waste dump to shovel	3 min
Load at shovel 1	3 min
Dump at crusher	1 min
Load at shovel 2	3 min
Dump at waste	1 min

Open Pit Truck/Shovel Haulage System Optimization (Optimal Route Selection), Four-node example, Closed out



Open Pit Truck/Shovel Haulage System Optimization (Optimal Route Selection), Four-node example, dispatched



Problem Statement

LP Model Objective Function is: (which automatically maximizes **synergy**)

Maximizing overall production rate which is equivalent to

(Maximizing both shovel and truck utilization (i.e: minimum shovel idle time and truck queue times)

Minimizing total number of trucks required for all shovel coverages without trucks queuing.

This is expressed as a combination of three components as follows:

- Total number of trucks on the road plus (+)
- Total number of trucks at the dumps plus (+)
- Total number of trucks at the shovels

SUBJECT TO:

- Truck balances around each node (i.e.: continuity and rate-limiting) and
- non-negativity

Assumptions of LP Model

1) based on
- shovel digging rates
- dump times
- travel times

Calculated in real time to generate
Optimal Routes for the current pit configuration

2) pit is viewed as a fixed (at any snapshot-point in time) number of muck points (sources) and damp points (sink) that are called nodes.
3) The nodes are connected by valid transactions routes that are called paths
4) some nodes are considered rate-limiting (e.g.: shovels and occasionally the crusher unless temporary ore stockpile is provided).
5) leach and waste dumps are assumed as capable of handling all transactions

MATHEMATICAL FORMULATION

Decision Variables for LP Model :

- x_{ij}: Average number of trucks per min over path i,j (trucks/min)
- t_{ij}: Average truck travel times in min over path i,j (min)
- t_i: number of minutes it takes to process a truck at node i (min)
- r_i: number of trucks processed at node i per min(trucks/min),
- $r_i = 1/ t_i$

Objective Function:

Min Z =

$$8x_{13} + 12x_{31} + 8x_{24} + 12x_{42} + 3x_{23} + 4x_{23} + t_1 x_{31} + t_2 x_{42} + 2$$

MATHEMATICAL FORMULATION Continued

Constraints:

- **For waste shovel (rate limiting node) ;**

$$x_{14} + x_{24} - x_{42} = 0$$

$$x_{42} - r_4 = 0 \qquad r_4 = 1/t_4$$

- **For ore shovel (rate limiting node);**

$$x_{13} + x_{23} - x_{31} = 0$$

$$x_{31} - r_3 = 0 \qquad r_3 = 1/t_3$$

MATHEMATICAL FORMULATION Continued

- **For waste dump (non-rate limiting node);**

$$x_{42} - x_{24} - x_{23} = 0$$

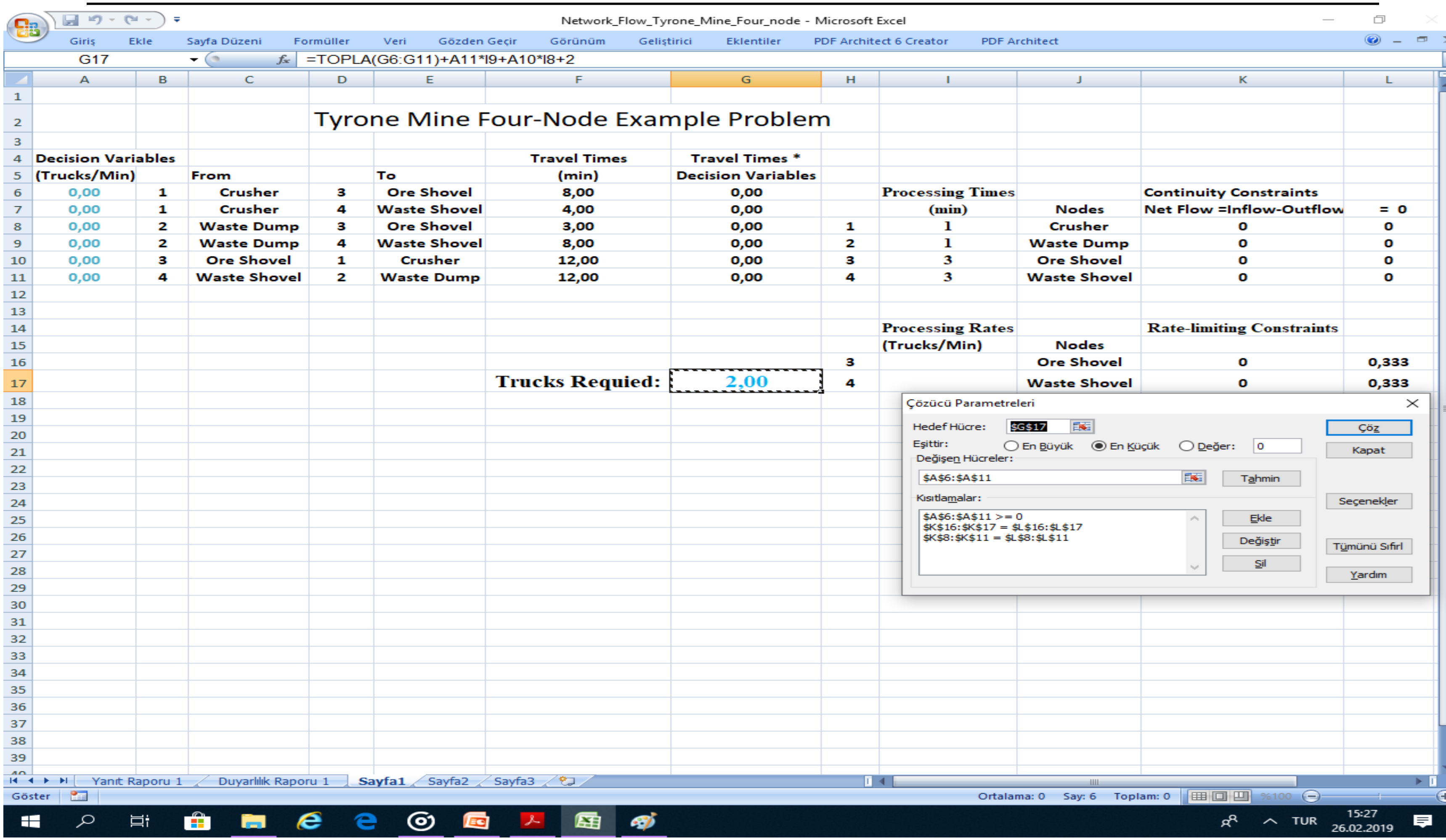
- **For crusher (non-rate limiting node)**

$$x_{31} - x_{13} - x_{14} = 0$$

- **Non-negativity;**

$x_{ij} \geq 0$ for all i and j

Excel Solver Model for Example Problem



Excel Solver Output for Example Problem

Network_Flow_Tyrone_Mine_Four_node - Microsoft Excel

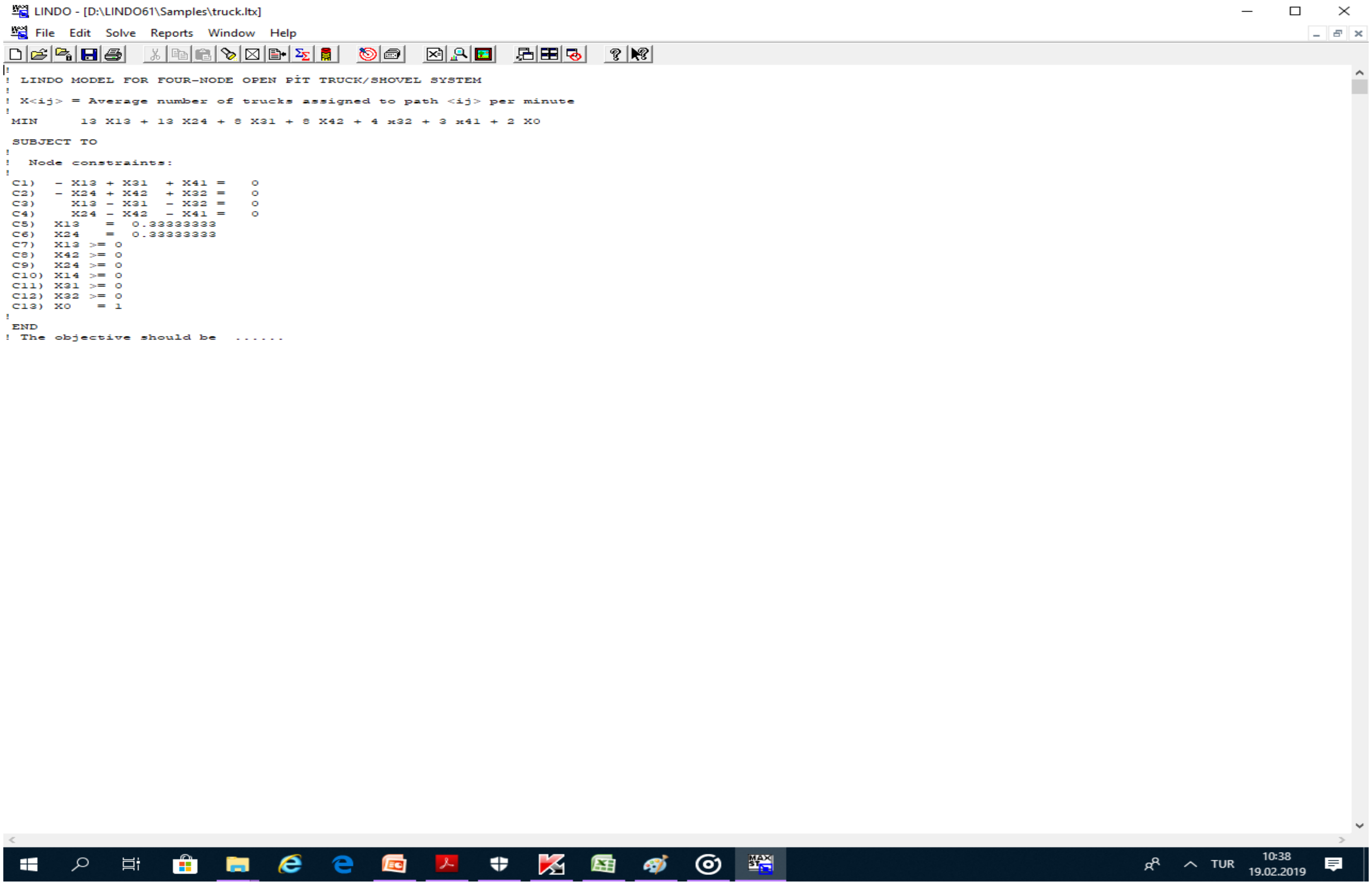
Tyrone Mine Four-Node Example Problem

Decision Variables (Trucks/Min)		From		To	Travel Times (min)	Travel Times * Decision Variables					
0,00	1	Crusher	3	Ore Shovel	8,00	0,00		Processing Times		Continuity Constraints	
0,33	1	Crusher	4	Waste Shovel	4,00	1,33		(min)	Nodes	Net Flow =Inflow-Outflow	= 0
0,33	2	Waste Dump	3	Ore Shovel	3,00	1,00	1	1	Crusher	0	0
0,00	2	Waste Dump	4	Waste Shovel	8,00	0,00	2	1	Waste Dump	0	0
0,33	3	Ore Shovel	1	Crusher	12,00	4,00	3	3	Ore Shovel	0	0
0,33	4	Waste Shovel	2	Waste Dump	12,00	4,00	4	3	Waste Shovel	0	0
								Processing Rates		Rate-limiting Constraints	
								(Trucks/Min)	Nodes		
							3		Ore Shovel	0	0,333
					Trucks Requied:	13,00	4		Waste Shovel	0	0,333

Yanıt Raporu 1 | Duyarlılık Raporu 1 | Sayfa1 | Sayfa2 | Sayfa3

Hazır — Ortalama: 0,22 Say: 6 Toplam: 1,33

LINDO Program for Four-Node Tyrone Mine Truck/Shovel Optimization Problem


LINDO - [D:\LINDO61\Samples\truck.ltx]

File Edit Solve Reports Window Help

```
!
! LINDO MODEL FOR FOUR-NODE OPEN PİT TRUCK/SHOVEL SYSTEM
!
! X<ij> = Average number of trucks assigned to path <ij> per minute
!
MIN     13 X13 + 13 X24 + 8 X31 + 8 X42 + 4 x32 + 3 x41 + 2 X0

SUBJECT TO
!
!  Node constraints:
!
C1)   - X13 + X31  + X41 =    0
C2)   - X24 + X42  + X32 =    0
C3)     X13 - X31  - X32 =    0
C4)     X24 - X42  - X41 =    0
C5)   X13    =  0.33333333
C6)   X24    =  0.33333333
C7)   X13 >= 0
C8)   X42 >= 0
C9)   X24 >= 0
C10)  X14 >= 0
C11)  X31 >= 0
C12)  X32 >= 0
C13)  X0   = 1
!
END
! The objective should be  ......
```

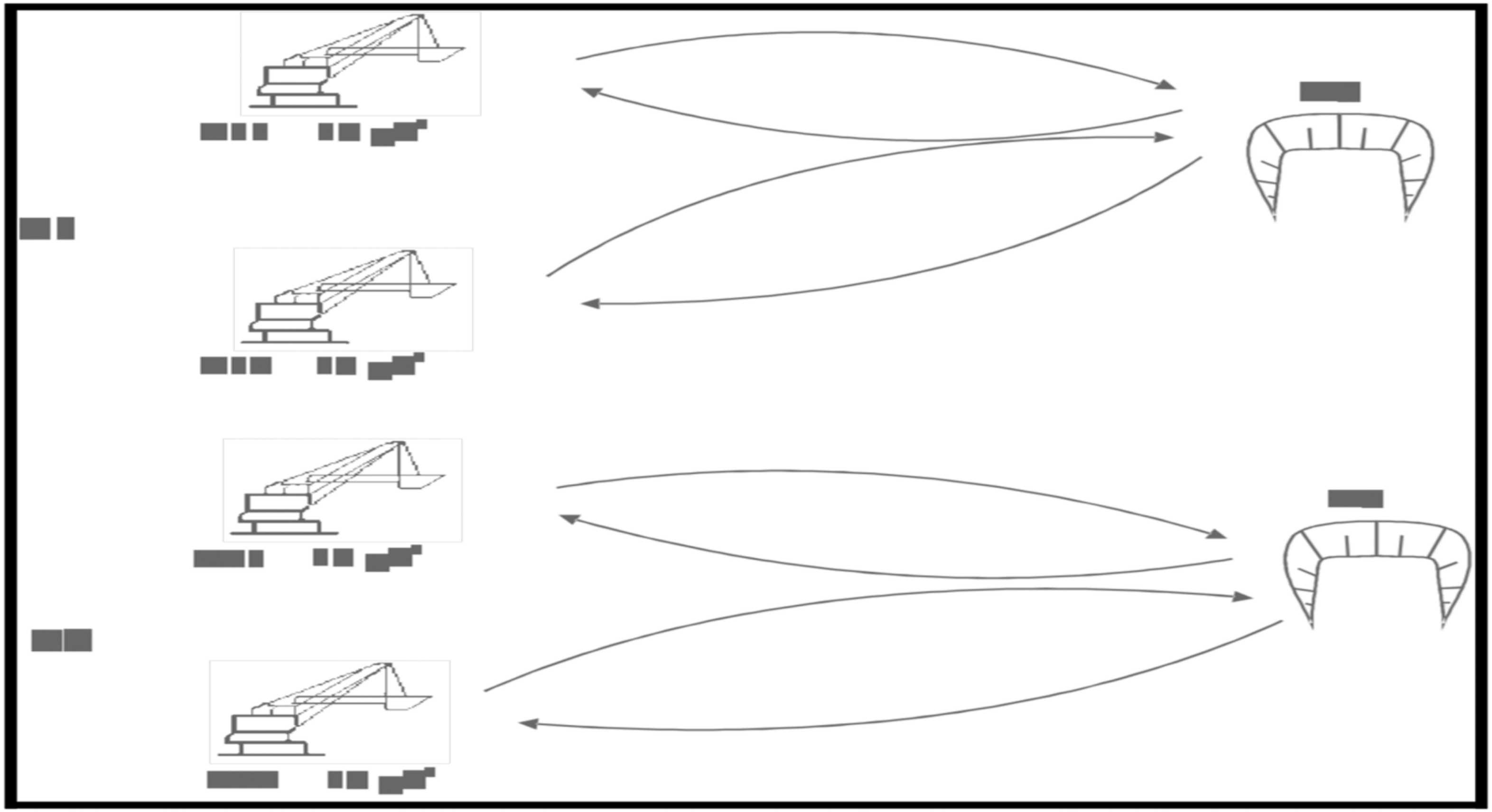
LINDO Output for Four-Node Tyrone Mine Truck/Shovel Optimization Problem

```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      13.00000

 VARIABLE        VALUE          REDUCED COST
      X13        0.333333          0.000000
      X24        0.333333          0.000000
      X31        0.000000          4.000000
      X42        0.000000          5.000000
      X32        0.333333          0.000000
      X41        0.333333          0.000000
       X0        1.000000          0.000000
      X14        0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
      C1)        0.000000          0.000000
      C2)        0.000000          0.000000
      C3)        0.000000          4.000000
      C4)        0.000000          3.000000
      C5)        0.000000        -17.000000
      C6)        0.000000        -16.000000
      C7)        0.333333          0.000000
      C8)        0.000000          0.000000
      C9)        0.333333          0.000000
     C10)        0.000000          0.000000
     C11)        0.000000          0.000000
     C12)        0.333333          0.000000
     C13)        0.000000         -2.000000

NO. ITERATIONS=       1


RANGES IN WHICH THE BASIS IS UNCHANGED:

                         OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
     X13       13.000000        INFINITY         INFINITY
     X24       13.000000        INFINITY         INFINITY
     X31        8.000000        INFINITY         4.000000
     X42        8.000000        INFINITY         5.000000
     X32        4.000000        4.000000         INFINITY
     X41        3.000000        5.000000         INFINITY
      X0        2.000000        INFINITY         INFINITY
     X14        0.000000        INFINITY         0.000000

                         RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS          INCREASE         DECREASE
      C1        0.000000        0.000000         0.000000
      C2        0.000000        0.000000         0.000000
      C3        0.000000        0.000000         0.000000
      C4        0.000000        0.000000         0.000000
      C5        0.333333        0.000000         0.000000
      C6        0.333333        0.000000         0.000000
      C7        0.000000        0.333333         INFINITY
      C8        0.000000        0.000000         INFINITY
      C9        0.000000        0.333333         INFINITY
     C10        0.000000        0.000000         INFINITY
     C11        0.000000        0.000000         INFINITY
     C12        0.000000        0.333333         INFINITY
     C13        1.000000        INFINITY         1.000000
```

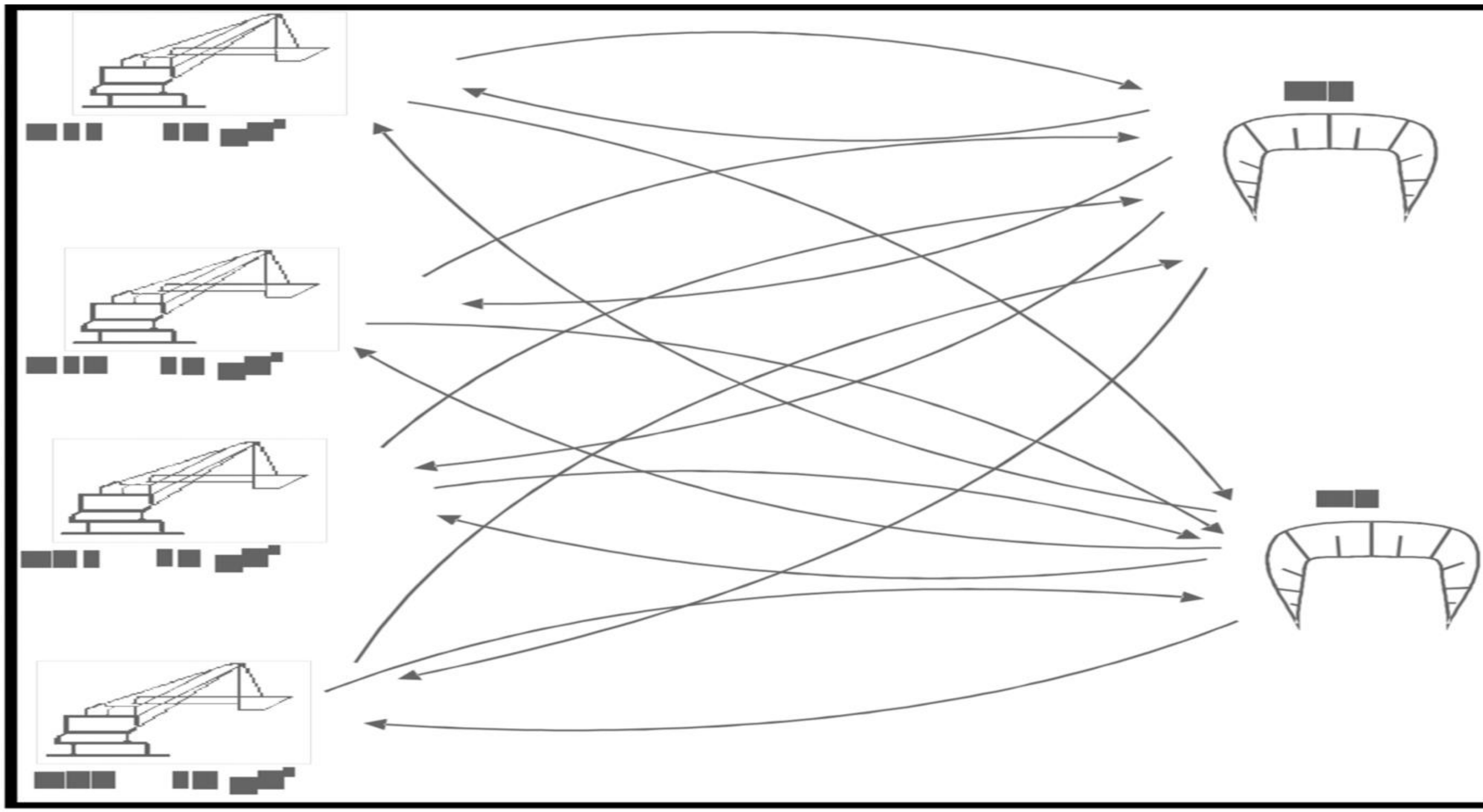
Input Data for Case Study Problem

Lengths and travelling times for possible paths			
Path	Path length (m)	Travel loaded (min)	Travel empty (min)
S11-W5	780	2.5	1.5
S11-W6	1205	5.4	3.9
S12-W5	2615	6.5	4.6
S12-W6	1068	4.7	3.0
S21-W5	1500	6.0	4.6
S21-W6	1874	8.0	5.3
S22-W5	1337	5.7	4.2
S22-W6	1753	7.5	5.0

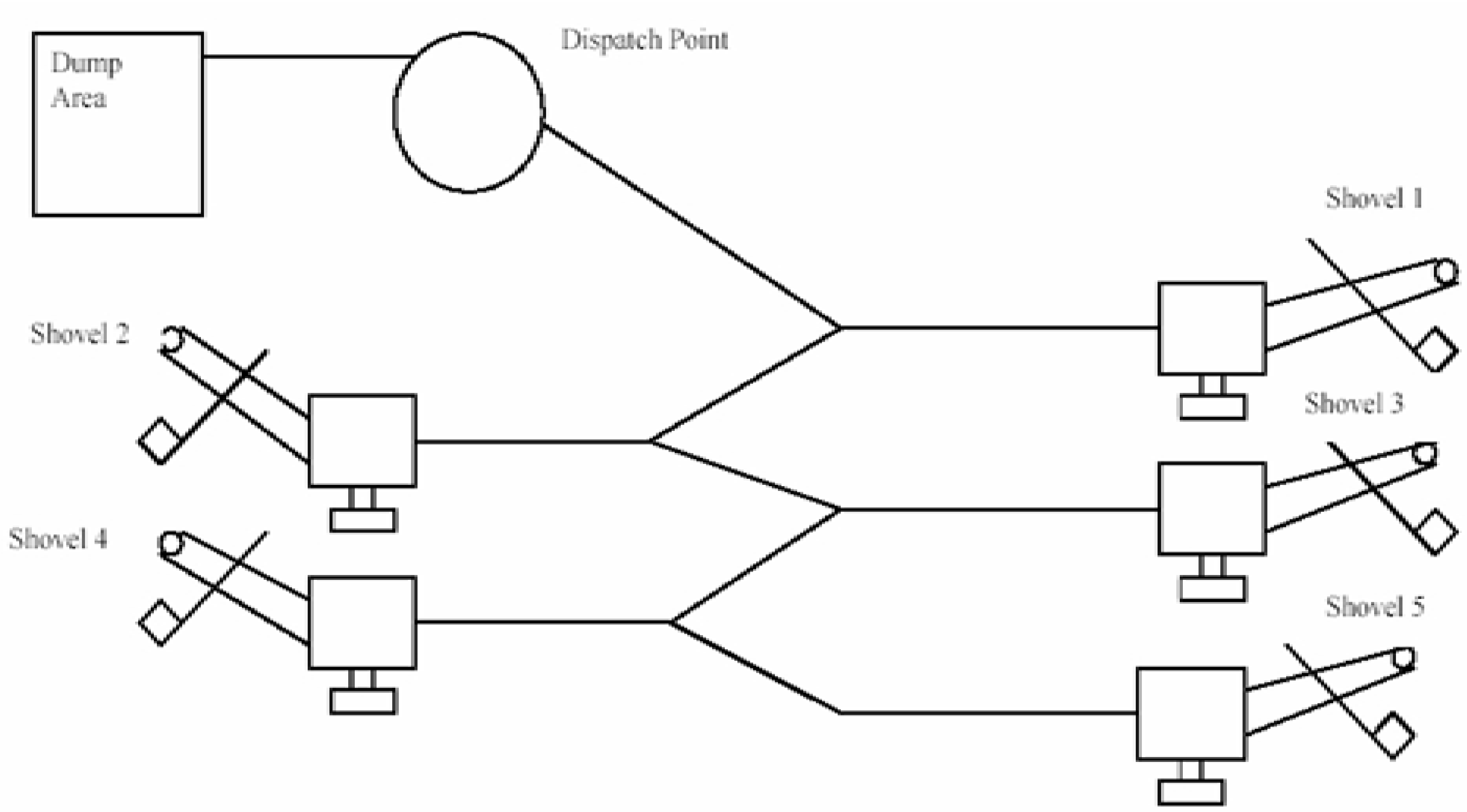
Shovel-Truck System as Closed-Out System



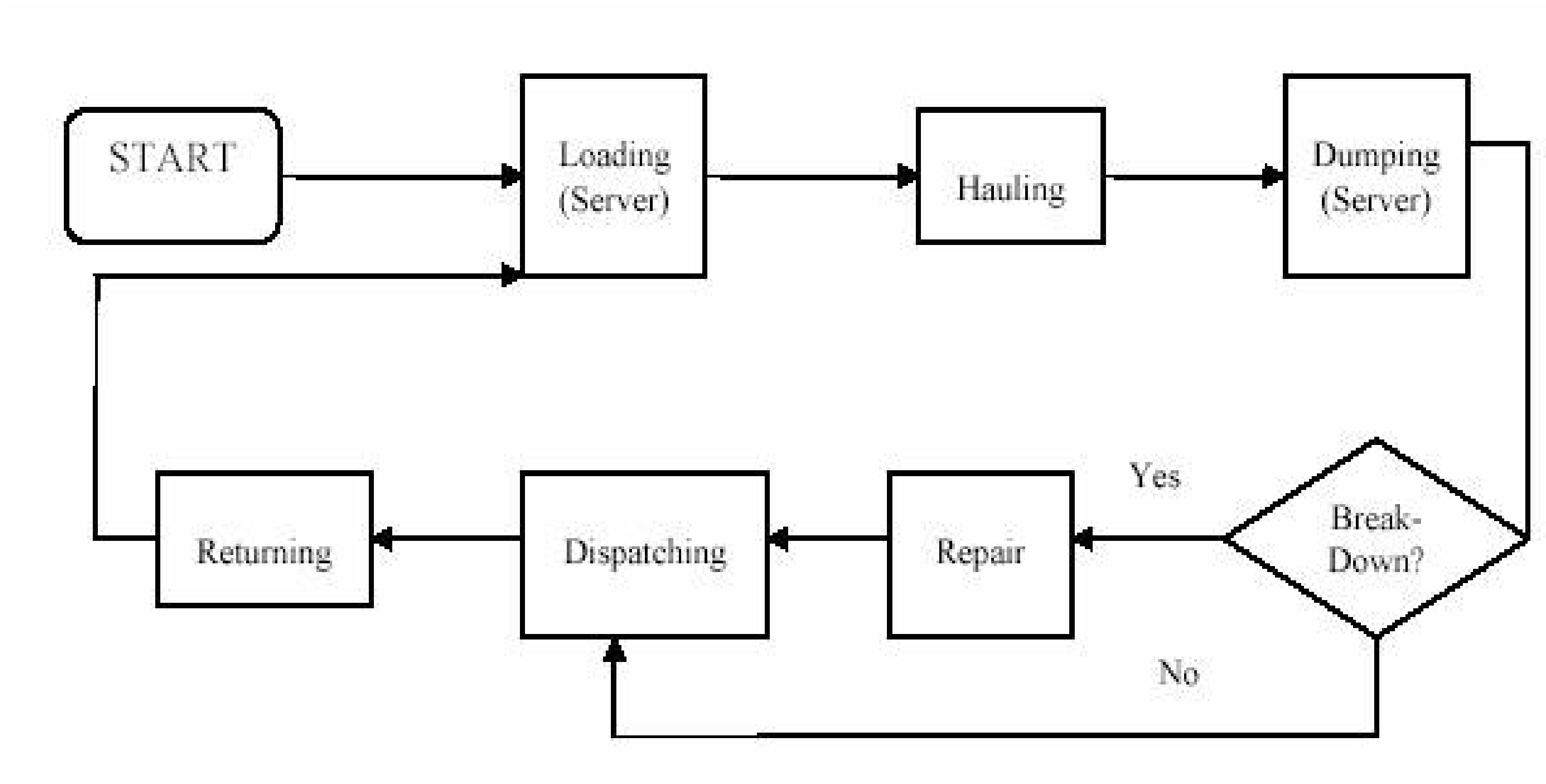
All possible Truck Paths



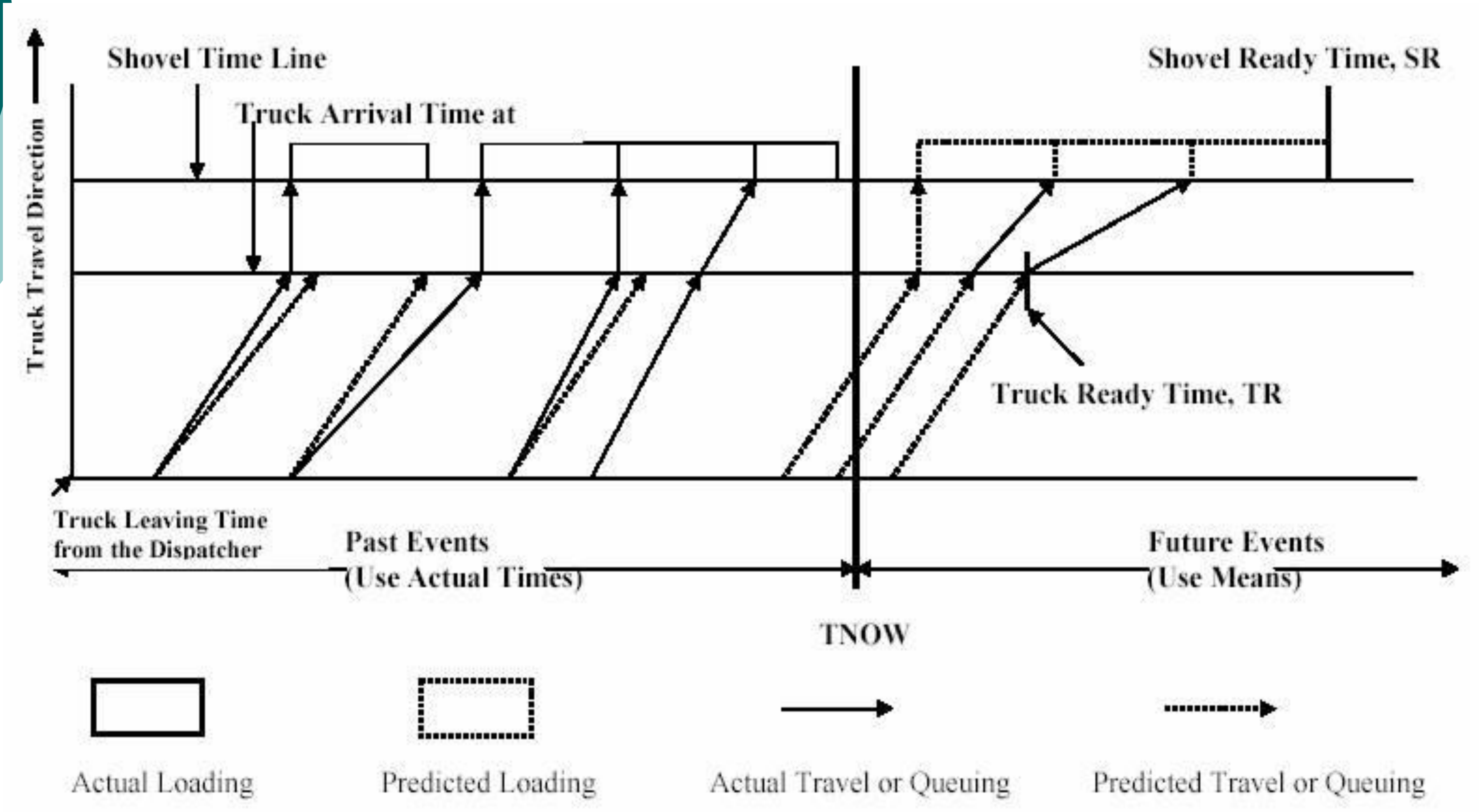
A Typical Truck-Shovel Mining System



Truck-Shovel Modeling Concept
Discrete-Event-System Simulation Approach



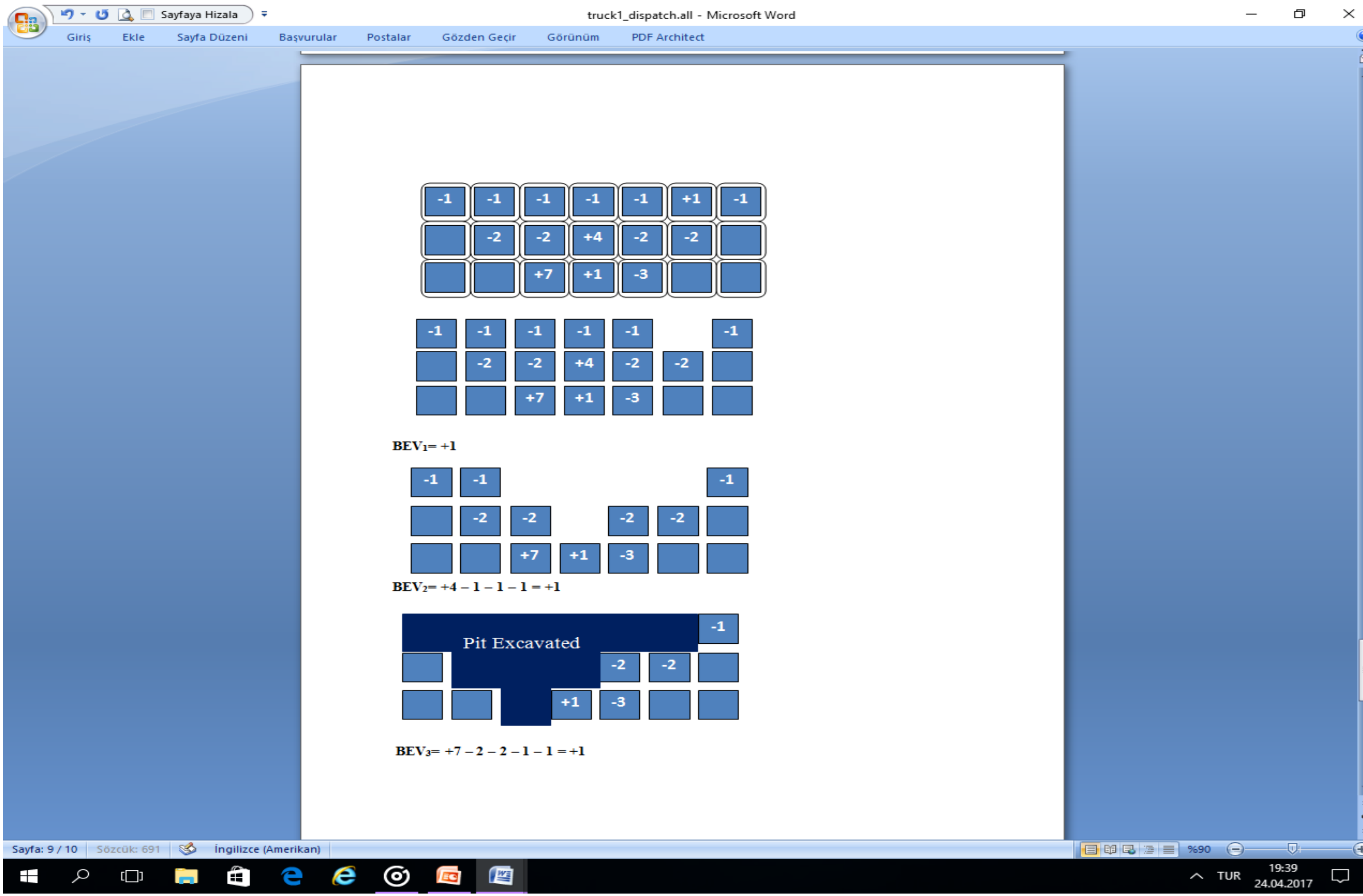
An Example Shovel Loading Gantt Chart



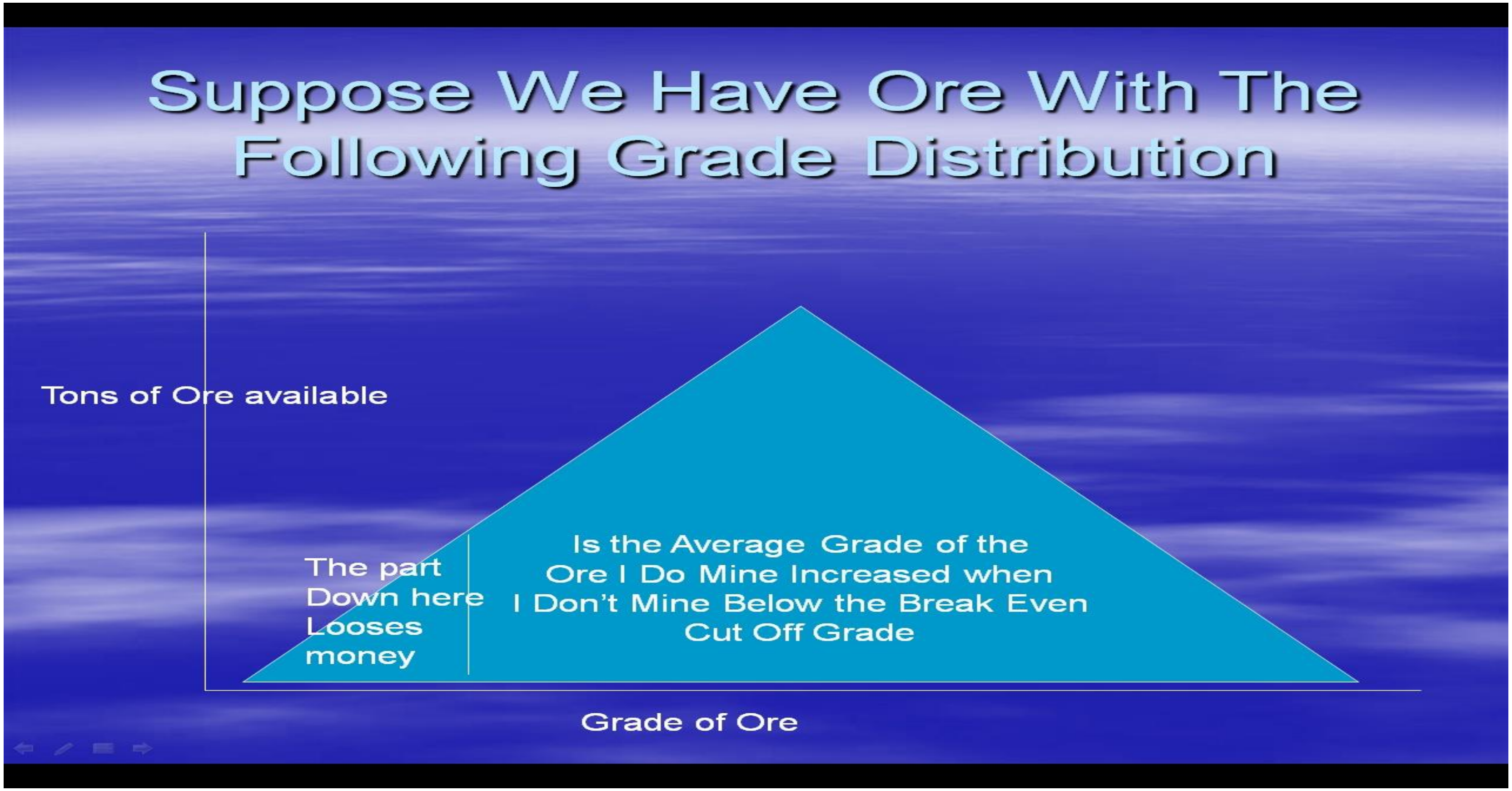
Example Problem2:Ultimate Pit Limit Design by Positive Moving Cone Algorithm

-1	-1	-1	-1	-1	+1	-1
	-2	-2	+4	-2	-2	
		+7	+1	-3		

Ultimate Pit Limit Design by Positive Moving Cone Algorithm Con't



Example Problem3: Cutoff Grade Optimization Problem



Example Problem3: Cutoff Grade Optimization Problem Con't

Grade Category Mid Point (ons/ton)	Grade Category (ons/ton)	Tonnage×100 (ton)
0,001	0 - 0,02	70000
0,0225	0,02 - 0,025	7257
0,0275	0,025 - 0,030	6319
0,0325	0,030 - 0,035	5591
0,0375	0,035 - 0,040	4598
0,0425	0,040 - 0,045	4277
0,0475	0,045 - 0,050	3465
0,0525	0,050 - 0,055	2428
0,0575	0,055 - 0,060	2307
0,0625	0,060 - 0,065	1747
0,0675	0,065 - 0,070	1640
0,0725	0,070 - 0,075	1485
0,0775	0,075 - 0,080	1227
0,0090	0,080 - 0,1	3598
0,229	0,1 - 0,358	9576

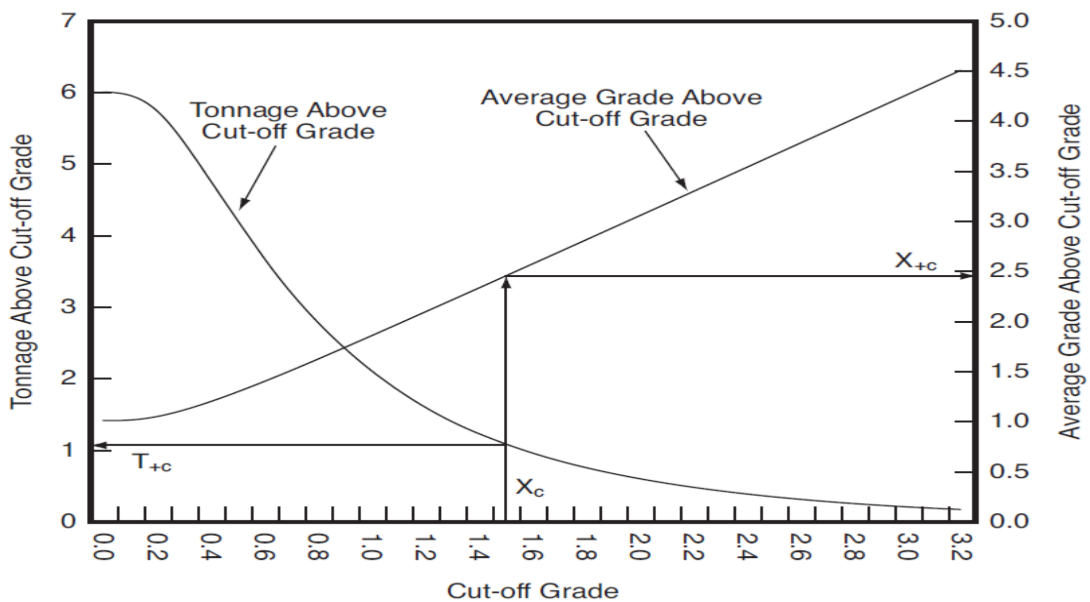
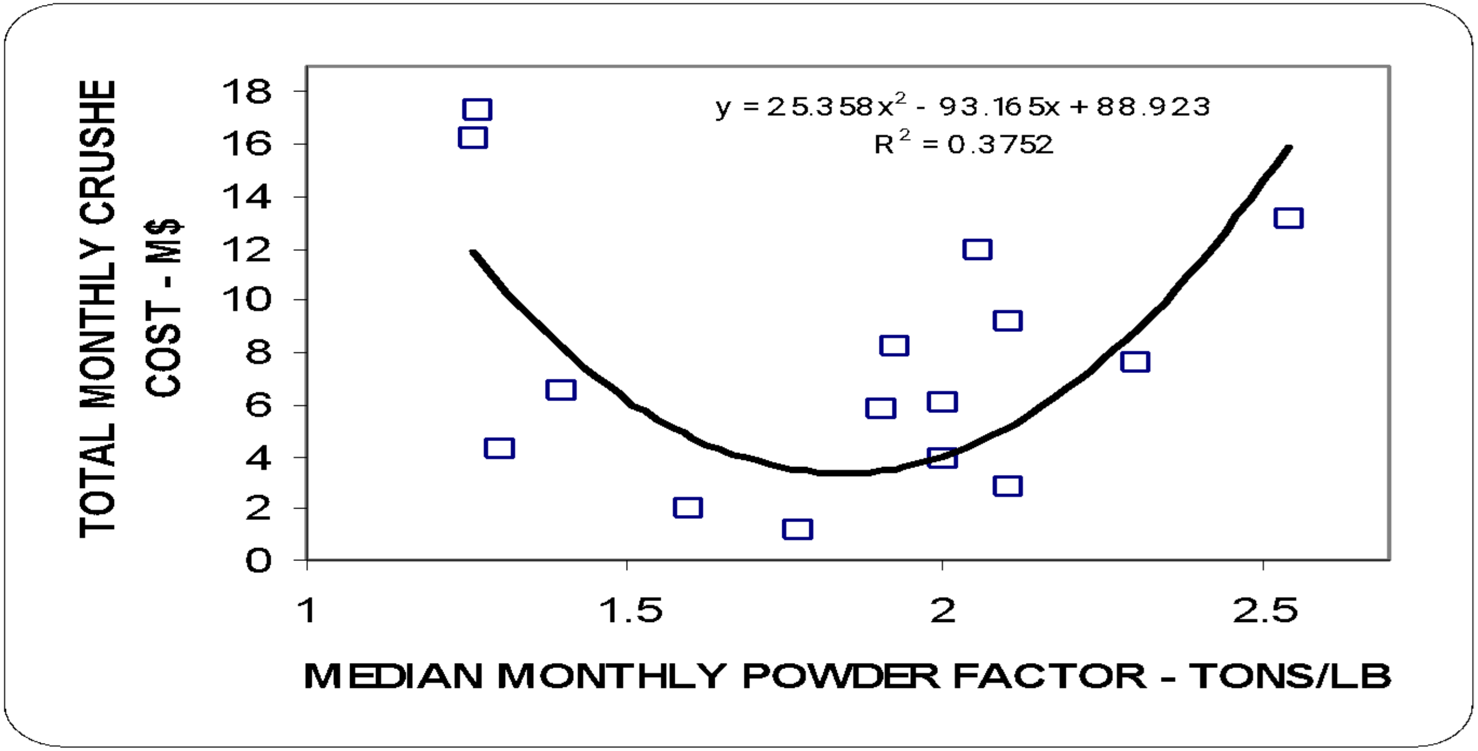


FIGURE 2-1 Example of grade–tonnage curve

Cutoff Grade Optimization Problem Con’t

Grade Category Mid-Point (ons/ton)	Grade Category (ons/ton)	Tonnage×100 (ton)
0,001	0 - 0,02	70000
0,0225	0,02 - 0,025	7257
0,0275	0,025 - 0,030	6319
0,0325	0,030 - 0,035	5591
0,0375	0,035 - 0,040	4598
0,0425	0,040 - 0,045	4277
0,0475	0,045 - 0,050	3465
0,0525	0,050 - 0,055	2428
0,0575	0,055 - 0,060	2307
0,0625	0,060 - 0,065	1747
0,0675	0,065 - 0,070	1640
0,0725	0,070 - 0,075	1485
0,0775	0,075 - 0,080	1227
0,0090	0,080 - 0,1	3598
0,229	0,1 - 0,358	9576
Total		**125000**

Optimization of Blasting Design



Activity Diagram for Strategic Planning

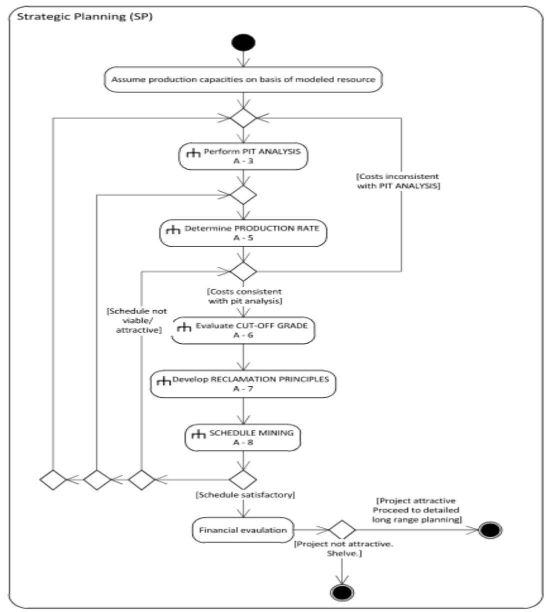


Figure 3-5: Conventional planning activity diagram-strategic planning

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Activity Diagram for Short-Range Planning

Short-range planning is a constant activity during the production stage of a mining project's life. Large mines will have planners on site seven days a week to respond to changes as they occur. The relationship between the different planning stages is shown in Figure 3-20.

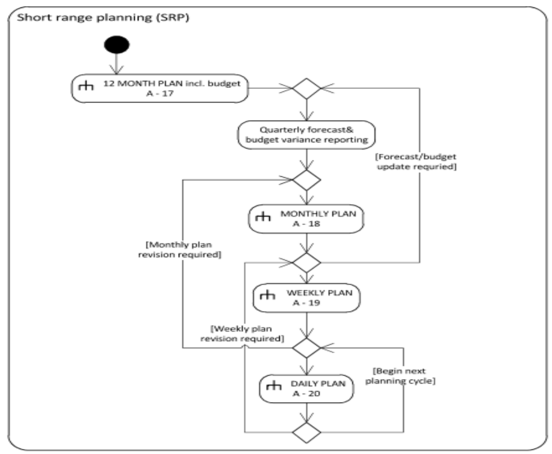

Figure 3-20: Conventional planning activity diagram – short range planning

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IBM_wp_envisioning_mining - Microsoft Word
1. People and Work
2. Sustainability
Collaboration
Governance and Workforce
Safety
Business Model Innovation
Energy and Environment
The Future of Mining
Asset Management
Remote Operations
Productivity, efficiency & cost reduction
Information integration and visualization
3. Operations and Technology
Figure 1: Strategic Areas of Focus for the Future of Mining
IBM Global Business Services 5

Thank you for your Attention.